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Preferences for Redistribution in an Aging Economy

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Abstract

It is one of the controversial issues whether and to what extent population aging affects the size of redistribution. Introducing some kind of altruism into a simple overlapping generations model with endogenous labor supply, we examine the relation between population aging and a redistributive tax rate in a voting equilibrium. We can observe the relation tends to be negative in models of intergenerational altruism, but the relation is not monotonic but complex in many cases. It suggests the puzzle of Razin et-al. (2002) may be attributed to lack of information about individual preferences for redistribution.

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1 Introduction

It has been one of the most controversial issues especially in the research field of public economics and population economics whether and to what extent population aging (an increase in the old-age dependency ratio) affects the size of the welfare state since Razin et-al. (2002) showed the increased size of the nonworking population may well lead to lower taxes and transfers.[1]

The mechanism is simple. First, population aging makes the political power of older people stronger, which encourages redistribution because they are net beneficiaries of the related public policies. Second, higher taxes deteriorate economic efficiency associated with labor supply (Meltzer and Richard (1981)), human capital investment (Razin et-al. (2002)), or fertility (Hirazawa et-al. (2010)). This economic effect discourages redistribution. Moreover, the effect becomes larger if young workers expect current distribution policies to be kept in the future because they are not only net contributors now but also net beneficiaries in the future.[2]

In addition to the political and economic effects on redistribution, some researchers examine another factors of individual preferences for redistribution. In an empirical analysis, Corneo and Grüner (2002) shows not only 'homo oeconomicus effect' but also 'the public values effect' and 'the social rivalry effect' play a significant role in shaping individual preferences for redistribution in some countries. In a theoretical analysis, Galasso (2003) shows, by introducing fair agents into the model of Meltzer and Richard (1981), a rise in the income inequality between poor and middle class may not decrease redistribution. Further research would be necessary to solve the puzzle of Razin et-al. (2002).

To make our model as simple as possible, we introduce some kind of altruism inferred from Corneo and Grüner (2002) into a two-period overlapping generations model with endogenous labor supply such as Meltzer and Richard (1981). In each period individuals enter the economy with different ability. They work in the first period and retire in the second period. In each period voters consist of the younger and older generation. They vote on the size of unfunded public pensions.

We assume three types of altruism.[3] First, individuals with ability w in one generation care about the welfare of individuals with ability w in the other generation. This assumption reflects the conscious of family background or social class, or the rivalry among heterogenous groups. We call it 'the ability-biased intergenerational altruism'. Second, individuals in one generation care about a social welfare of the other generation. Since the altruism is not related to ability, we call it 'the ability-neutral intergenerational altruism'. Finally, individuals care about a social welfare of their own generation. It reflects the sympathy to-

[1] See Disney (2007), Galasso and Profeta (2007), Razin and Sadka (2007), Simonovits (2007), Shelton (2008), and Hirazawa et-al. (2010) about the issue.

[2] Alesina and La Ferrara (2005) shows support for redistributive policies is negatively affected by the likelihood of moving above an income threshold that is likely to separate the winners and the losers from redistribution. Borck (2007) shows individuals with higher income have an incentive to vote on a larger size of redistribution because they live longer in the retirement than the lower income group. In a model of endogenous fertility, Hirazawa et-al. (2010) shows population aging (a decrease in the adult mortality rate) may result in a higher contribution rate and a higher fertility rate.

[3] Assuming within-group altruism (fractionalization) and/or between-group altruism (antagonism), Lind (2007) shows that the former reduces the support for redistribution and the latter has the opposite effect.

ward the same generation or the rivalry between different generations. We call it 'intragenerational altruism'.

Numerical analysis shows that population aging is likely to decrease redistribution in (i) the ability-biased intergenerational altruism model and (ii) the ability-neutral intergenerational altruism model. Interestingly, the majority in (ii) consists of the older generation. If the median voter belongs to the lower income group in the younger generation, population aging increases redistribution. We also observe that the relation between population aging and the size of redistribution is not monotonic but complex in many cases. It suggests the puzzle of Razin et-al. (2002) may be attributed to lack of information about individual preferences for redistribution as Corneo and Grüner (2002) insists.

The structure of the paper is as follows. In section 2 we first setup the basic model and then introduce three types of altruism into the basic model. In section 3 we present numerical examples. The final section concludes the paper.

2 The model

2.1 Basic model

We use a simple two-period overlapping generations model. In each period, individuals enter the economy with different ability $w \in W$. The probability distribution function is $F(w) = \int f(w)dw$, where $f(w) \geq 0$ is a density function.

Individuals are risk-neutral with respect to consumption. A gross interest rate is normalized to one, and a gross rate of population growth, $n > 0$, is constant over time. The old-age dependency ratio is $1/(1+n)$, which increases when n decreases. If $n > 1$, then unfunded public pensions are more efficient than funded pensions.

The optimization problem for an individual with ability w who is born in period t is

$$\max \quad U_t = c_t - u(l_t)$$

subject to the lifetime budget constraint,

$$(1 - \tau_t)wl_t + P_{t+1} = c_t$$

where c_t and l_t stand for present-valued consumption and working time, respectively. The marginal disutility of labor is positive and increasing ($u' > 0$ and $u'' > 0$). $\tau_t \in [0, 1)$ is a tax rate, and P_{t+1} is the pension benefit in the retirement.

The first-order condition requires

$$u'(l_t) = (1 - \tau_t)w$$

which gives a labor supply function, $l_t = l(w_t, \tau_t)$.

The balanced budget for the unfunded public pension requires

$$P_t = n\tau_t\bar{y}(\tau_t) \tag{1}$$

where we define the labor income of an individual with ability w by $y(w, \tau_t) = wl(w, \tau_t)$, and the average labor income by $\bar{y}(\tau_t)$,

$$\bar{y}(\tau_t) = \int_W y(w, \tau_t)dF(w)$$

The indirect utility of an individual with ability w is given by

$$U(w, \tau_t, \tau_{t+1}) = (1-\tau_t)y(w,\tau_t) - u(l(w,\tau_t)) + n\tau_{t+1}\bar{y}(\tau_{t+1}) \qquad (2)$$

We assume each individual in the younger generation expects the tax rate in his second period is the same as that in this period, $\tau_{t+1} = \tau_t$. For a notational simplicity, we omit time subscripts hereafter.

In each period, an individual with ability w in the younger generation prefers a tax rate which maximizes (2). Denote it by $\tau^y(w)$. An individual with ability w in the older generation prefers a tax rate which maximizes (1). Denote it by $\tau^o(w)$. Without altruism, the preferred tax rate for the older generation is independent of ability, $\tau^o(w) = \tau^o$. However, it does depend on ability if we introduce ability-biased intergenerational altruism below.

With the envelope theorem, we have

$$\frac{\partial U}{\partial \tau} = -y(w,\tau) + n\bar{y}(\tau)\left[1-\sigma(\tau)\right] \qquad (3)$$

where $\sigma(\tau)$ stands for the tax elasticity of the average income,

$$\sigma(\tau) \equiv -\frac{\tau}{\bar{y}}\frac{\partial \bar{y}}{\partial \tau}$$

Assumption 1. The tax elasticity of the average income is increasing, that is,

$$\sigma'(\tau) > 0 \qquad (4)$$

for any $\tau \in [0,1)$.

Since $\partial^2 U/(\partial\tau\partial w) < 0$, $\tau^y(w)$ is decreasing in w. In general, $\partial^2 U/(\partial\tau\partial n) > 0$ because a higher n makes unfunded public pensions more efficient. It suggests the preferred tax rate $\tau^y(w)$ shifts downward when n decreases.

The optimal tax rate for the older generation is given by

$$\sigma(\tau^o) = 1 \qquad (5)$$

Equation (5) implies all the individuals in the older generation prefer a tax rate which maximizes the pension benefit. We know $\tau^y(w) \leq \tau^o$ for any $w \in W$ because $(\partial U/\partial\tau)_{\tau=\tau^o} = -y(w,\tau^o) \leq 0$. The voting equilibrium in the basic model is given by

$$\tau^* = \begin{cases} \tau^y(w_m^y) & \text{if} \quad n > 1 \\ \tau^o & \quad\;\; n < 1 \end{cases}$$

where the median voter in the younger generation is given by

$$F(w_m^y) = \frac{1}{2}\left(1 - \frac{1}{n}\right)$$

Population aging (an increase in the old-age dependency ratio) affects the equilibrium in two ways. One is a political effect. The median voter moves to lower income group, which increases the tax rate. The other is an economic effect. The preferred tax rate for the median voter decreases because unfunded public pensions become less efficient.

In the following subsections, we introduce some kind of altruism into the basic model to examine ‘the public value effect’ and ‘the social rivalry effect’ in Corneo and Grüner (2002).

2.2 Ability-biased intergenerational altruism

In this subsection, we assume individuals with ability w in one generation care about individuals with w in the other generation. The intergenerational altruism is ability-biased. The positive correlation of ability between the generations reflects the conscious of family background or social class, or the rivalry among heterogenous groups.

The objective function of an individual w in the younger generation is

$$W_1^y(w,\tau) = U(w,\tau) + \rho^y P(\tau) \tag{6}$$

where $\rho^y \geq 0$ stands for an altruism parameter of the younger generation.

The objective function of an individual w in the older generation is

$$W_1^o(w,\tau) = P(\tau) + \rho^o U(w,\tau) \tag{7}$$

where $\rho^o \geq 0$ stands for an altruism parameter of the older generation.

We have

$$\frac{\partial W_1^y}{\partial \tau} = -y(w,\tau) + (1+\rho^y)n\bar{y}(\tau)[1-\sigma(\tau)] \tag{8}$$

$$\frac{\partial W_1^o}{\partial \tau} = -\rho^o y(w,\tau) + (1+\rho^o)n\bar{y}(\tau)[1-\sigma(\tau)] \tag{9}$$

which give the preferred tax rates, $\tau^y(w)$ and $\tau^o(w)$.

The voting equilibrium is given by

$$\begin{aligned} \tau^* &= \tau^y(w_m^y) = \tau^o(w_m^o) \\ nF(w_m^y) + F(w_m^o) &= \frac{1}{2}(1+n) \end{aligned}$$

[Figure 1 is here]

Figure 1 shows the voting equilibrium in the model of ability-biased intergenerational altruism. The vertical axis measures the tax rate, and the horizontal axis measures the ability of the younger generation rightward from the origin, and the ability of the older generation leftward from the origin. The dashed curve stands for the preferred tax rate in the basic model, and the solid curve stands for the preferred tax rate in the model of ability-biased intergenerational altruism. Comparing to the basic model, the tax curve for the younger generation shifts upward, and the tax curve for the older generation shifts downward because both generations care about each other. Individuals with higher ability in the older generation prefer lower tax rates because they care about individuals with higher ability in the younger generation who prefer lower tax rates. There exist a marginal voter in the younger generation, w_m^y, and a marginal voter in the older generation, w_m^o, such that the lower ability group in the younger generation, $w^y \leq w_m^y$, and in the older generation, $w^o \leq w_m^o$, form the majority. In section 3, we examine the effect of population aging on the the equilibrium tax rate numerically.

2.3 Ability-neutral intergenerational altruism

In this subsection, we assume individuals in one generation care about a social welfare of the other generation. The intergenerational altruism is ability-neutral.

The objective function of an individual w in the younger generation is the same as (6), $W_2^y(w,\tau) = U(w,\tau) + \rho^y P(\tau)$.

The objective function of an individual w in the older generation is

$$W_2^o(\tau) = P(\tau) + \rho^o \int_W U(w,\tau) dF(w) \tag{10}$$

Differentiating equation (10) with respect to τ, we have

$$\frac{\partial W_2^o}{\partial \tau} = \bar{y}(\tau) \left[(1+\rho^o) n \left(1 - \sigma(\tau)\right) - \rho^o \right]$$

which gives the optimal rate for the older generation such as

$$\sigma(\tau^o) = 1 - \frac{\rho^o}{n(1+\rho^o)} \tag{11}$$

Without altruism, τ^o is the same as the basic model. τ^o is decreasing in ρ^o, and increasing in n given that $\rho^o > 0$. If the altruism is stronger, the older generation agree on less redistribution. If the population growth rate decreases, they also agree on less distribution because the decreased efficiency of unfunded pensions makes the younger generation worse-off.

[Figure 2 is here]

Figure 2 shows the voting equilibrium in the model of ability-neutral intergenerational altruism. The tax curve for the younger generation is the same as Figure 1. Contrary to the ability-biased case, the tax curve for the older generation shifts downward in parallel. Thus, one of the possible equilibria in which the older generation form the majority, τ^o, is politically stable because the other equilibria could be realized only if some group in the younger generation form the majority.

Specifically, denote by $\hat{w}^y$ the ability of individuals in the younger generation who would prefer the same tax rate as the older generation. First, assume that $nF(\hat{w}^y) < (1+n)/2$ and $n[1 - F(\hat{w}^y)] < (1+n)/2$. The voting equilibrium is τ^o because the younger generation cannot form a majority group. Second, if $nF(\hat{w}^y) > (1+n)/2$ or if $n[1-F(\hat{w}^y)] > (1+n)/2$, then the younger generation can form the majority. In the former case, the tax rate, τ_H, is high because the lower ability group becomes the majority. In the latter case, the tax rate, τ_L, is low because the higher ability group does.

The voting equilibrium in the ability-neutral intergenerational altruism model is given by

$$\tau^* = \begin{cases} \tau_L & \\ \tau^o & \text{if} \\ \tau_H & \end{cases} \quad \begin{array}{c} F(\hat{w}^y) < \frac{1}{2}(1-\frac{1}{n}) \\ \frac{1}{2}(1-\frac{1}{n}) < F(\hat{w}^y) < \frac{1}{2}(1+\frac{1}{n}) \\ \frac{1}{2}(1+\frac{1}{n}) < F(\hat{w}^y) \end{array}$$

where $\tau^y(\hat{w}^y) = \tau^o$, and

$$\begin{aligned}\tau_H &= \tau^y\left(F^{-1}\left(\frac{n+1}{2n}\right)\right)\\ \tau_L &= \tau^y\left(F^{-1}\left(\frac{n-1}{2n}\right)\right)\end{aligned}$$

2.4 Intragenerational altruism

In this subsection, we assume individuals care about a social welfare of their own generation. It reflects the sympathy toward the same generation or the rivalry between different generations.

The objective function of an individual w in the younger generation is given by

$$W_3^y = U(w,\tau) + \rho^y \int_W U(w,\tau)dF(w) \tag{12}$$

The objective function of an individual w in the older generation is

$$W_3^o = (1+\rho^o)P(\tau) \tag{13}$$

which gives τ^o in the basic model.

From (12), we have

$$\frac{\partial W_3^y}{\partial \tau} = -y(w,\tau) + \bar{y}(\tau)\left[n(1+\rho^y)(1-\sigma(\tau)) - \rho^y\right] \tag{14}$$

which gives $\tau^y(w)$.

[Figure 3 is here]

Figure 3 shows the voting equilibrium in the intragenerational altruism model. Comparing to the basic model, individuals with higher ability in the younger generation prefer higher tax rates because they care about the welfare of the lower income group. The voting equilibrium is given by

$$\tau^* = \begin{cases} \tau^y(w_m^y) & \text{if} \quad n > 1 \\ \tau^o & \quad\quad\;\; n < 1 \end{cases}$$

where $F(w_m^y) = \frac{1}{2}\left(1 - \frac{1}{n}\right)$.

When $n > 1$, population aging could decrease the tax rate because the median voter who belongs to the lower income group in the younger generation may well prefer a lower tax rate by the intragenerational altruism.

3 Numerical analysis

In this section we present numerical examples because it is difficult to get general results. Assumptions may be unrealistic, but we hope the results would be insightful.

Assumption 2. The ability w is uniformly distributed in $[0, 1]$.[4]

[4] More realistic cases are analyzed in section 3.5.

Assumption 3. The disutility of labor is a quadratic function such as

$$u(l) = \frac{l^2}{2\beta}$$

where $\beta > 0$ stands for a parameter associated with the willingness to work.

The labor supply function is given by

$$l = \beta(1-\tau)w$$

which gives the labor income,

$$y = wl = \beta(1-\tau)w^2$$

and the average labor income,

$$\bar{y} = \int_0^1 y dw = \frac{1}{3}\beta(1-\tau)$$

The tax elasticity of the average income is

$$\sigma(\tau) = \frac{\tau}{1-\tau}$$

which satisfies Assumption 1.

3.1 Basic model

From equation (5), the optimal tax rate for the older generation is

$$\tau^o = \frac{1}{2}$$

The optimal tax rate for an individual w in the younger generation is given by[5]

$$\tau^y(w) = \begin{cases} \frac{n-3w^2}{2n-3w^2} & \text{if} \quad 0 \le w \le \sqrt{\frac{n}{3}} \\ 0 & \quad\quad \sqrt{\frac{n}{3}} \le w \le 1 \end{cases} \tag{15}$$

The voting equilibrium is $\tau^* = \tau^o$ for $n < 1$, and $\tau^* = \tau^y(w_m^y(n))$ for $n > 1$, where $w_m^y(n) = \frac{1}{2}\left(1 - \frac{1}{n}\right)$.

We can verify $w_m^y(n) < \sqrt{\frac{n}{3}}$ for any $n > 0$. Therefore, we have

$$\tau^y(w_m(n)) = \frac{n - \frac{3}{4}\left(1-\frac{1}{n}\right)^2}{2n - \frac{3}{4}\left(1-\frac{1}{n}\right)^2} \tag{16}$$

[Figure 4 is here]

Figure 4 illustrates the relation between τ^* and n in the basic model. The relation is not monotonic. For a larger n, population aging decreases the tax rate because the economic effect dominates the political effect. For a smaller n, the tax rate increases because the political effect becomes dominant.

[5] See Appendix for the derivation.

3.2 Ability-biased intergenerational altruism

The optimal tax rate for an individual w in the younger generation is given by[6]

$$\tau^y(w) = \begin{cases} \frac{n(1+\rho^y)-3w^2}{2n(1+\rho^y)-3w^2} & \text{if} \quad 0 \leq w \leq \sqrt{\frac{n}{3}(1+\rho^y)} \\ 0 & \quad \sqrt{\frac{n}{3}(1+\rho^y)} \leq w \leq 1 \end{cases} \tag{17}$$

The optimal tax rate for an individual w in the older generation is

$$\tau^o(w) = \begin{cases} \frac{n(1+\rho^o)-3\rho^o w^2}{2n(1+\rho^o)-3\rho^o w^2} & \text{if} \quad 0 \leq w \leq \sqrt{\frac{n}{3}(1+\frac{1}{\rho^o})} \\ 0 & \quad \sqrt{\frac{n}{3}(1+\frac{1}{\rho^o})} \leq w \leq 1 \end{cases} \tag{18}$$

The voting equilibrium is summarized in the following proposition.

Proposition 1 *In the ability-biased intergenerational altruism model, the voting equilibrium is given by*

$$\tau^* = \frac{n(1+\rho^y) - \frac{3}{4}\left[\frac{\rho(1+n)}{1+\rho n}\right]^2}{2n(1+\rho^y) - \frac{3}{4}\left[\frac{\rho(1+n)}{1+\rho n}\right]^2} \tag{19}$$

where

$$\rho \equiv \sqrt{\frac{\rho^o(1+\rho^y)}{1+\rho^o}} \tag{20}$$

stands for a strength of altruism.

The median voter in the younger generation, w_m^y, and the median voter in the older generation, w_m^o, are respectively given by

$$\begin{aligned} w_m^y &= \frac{\rho(1+n)}{2(1+\rho n)} \\ w_m^o &= \frac{1+n}{2(1+\rho n)} \end{aligned}$$

A larger ρ increases w_m^y, and decreases w_m^o. Given that $\rho < 1$, a larger n increases both w_m^y and w_m^o.

Proof. See Appendix. ■

[Figure 5a and 5b are here]

Figure 5a and 5b illustrate the relation between τ^* and n in the ability-biased intergenerational altruism model. Figure 2a is a case in which intergenerational altruism is weak (we assume $\rho^y = 0$ and $\rho^o = 0.01$, which gives $\rho = 0.0995$.), and Figure 2b is a strong case $\rho^y = \rho^o = 0.1$, and $\rho = 0.316$).

When the intergenerational altruism is weak, the relation is not monotonic. The reason is the same as the basic model. On the one hand, a decrease in n makes the public pension less efficient, which decreases the optimal tax rate.

[6] See Appendix.

On the other hand, a decrease in n shifts the median voter towards the lower income group. This effect increases the optimal tax rate.

When the intergenerational altruism is strong, the relation between τ^* and n is positive in a broader region. This result support for Razin et-al. (2002). The strength of altruism, ρ in equation (20), is a key factor. Specifically, we have the following proposition.

Proposition 2 *Under Assumption 2 and 3,*

(i) If $0 < \rho < 1/9$, the tax rate is increasing in $n \in (0, \underline{n}), (\overline{n}, \infty)$, and decreasing in $n \in (\underline{n}, \overline{n})$, where

$$\begin{aligned} \underline{n} &= \frac{1 - 3\rho - \sqrt{(1-\rho)(1-9\rho)}}{2\rho} \\ \overline{n} &= \frac{1 - 3\rho + \sqrt{(1-\rho)(1-9\rho)}}{2\rho} \end{aligned}$$

(ii) If $1/9 \leq \rho < 1$, the tax rate is increasing in n.

Proof. See Appendix. ■

3.3 Ability-neutral intergenerational altruism

The optimal tax rate for an individual with ability w in the younger generation is given by equation (17).

From equation (11), the optimal tax rate for the older generation is given by

$$\tau^o = \frac{n(1+\rho^o) - \rho^o}{2n(1+\rho^o) - \rho^o} \tag{21}$$

Given that $\rho^o > 0$, the optimal tax rate for the older generation is strictly smaller than the optimal tax rate for the lowest ability group in the younger generation, $\tau^o < \tau^y(0)$. Since $\tau^y(w)$ is decreasing in w, and $\tau^o > 0$, there exists a unique ability level which satisfies $\tau^y(w) = \tau^o$. Denote it by $\hat{w}^y$. From equations (17) and (21), we have

$$\hat{w}^y = \frac{\rho}{\sqrt{3}} \tag{22}$$

where ρ is given by equation (20).

Specifically, we have the following proposition.

Proposition 3 *The voting equilibrium in the ability-neutral intergenerational altruism model is*

$$\tau^* = \begin{cases} \tau_L & \hat{w}^y < \frac{1}{2}(1-\frac{1}{n}) \\ \tau^o \quad \text{if} & \frac{1}{2}(1-\frac{1}{n}) < \hat{w}^y < \frac{1}{2}(1+\frac{1}{n}) \\ \tau_H & \frac{1}{2}(1+\frac{1}{n}) < \hat{w}^y \end{cases}$$

where

$$\begin{aligned} \tau_H &= \tau^y\left(\frac{1}{2}\left(1+\frac{1}{n}\right)\right) \\ \tau_L &= \tau^y\left(\frac{1}{2}\left(1-\frac{1}{n}\right)\right) \end{aligned}$$

$\tau^y(w)$, τ^o, and $\hat{w}^y$ are respectively given by equations (17), (21), and (22).

[Figure 6 is here]

Figure 6 illustrates the relation between τ^* and n in the model of ability-neutral intergenerational altruism. We assume $\rho^y = \rho^o = 0.1$. The critical ability is $\hat{w}^y = 0.183$. It implies $\tau^* = \tau^o$ if $n < 1.577$, and $\tau^* = \tau_L$ if $n > 1.577$. In a region where the critical mass consists of a higher income group in the younger generation, the tax rate moves as the basic model. In the process of population aging, however, the political power of the older generation becomes strong, and finally becomes the majority. It is noteworthy that this political change does not increase the tax rate but decrease the tax rate because the older generation care about a social welfare of the younger generation.

3.4 Intragenerational altruism

The optimal tax rate for the older generation is given $\sigma(\tau) = 1$, that is,

$$\tau^o = \frac{1}{2}$$

The optimal tax rate for an individual w is given by[7]

$$\tau^y(w) = \begin{cases} \frac{n(1+\rho^y)-\rho^y-3w^2}{2n(1+\rho^y)-\rho^y-3w^2} & \text{if} \quad 0 \leq w \leq \sqrt{\frac{(1+\rho^y)n-\rho^y}{3}} \\ 0 & \quad\ \ \sqrt{\frac{(1+\rho^y)n-\rho^y}{3}} \leq w \leq 1 \end{cases} \tag{23}$$

τ^y shifts downward when n decreases as the basic model. Introducing the intragenerational altruism makes the tax rate flat. Specifically, we have

$$\frac{\partial \tau^y}{\partial \rho^y} \gtreqless 0 \Leftrightarrow w^2 \gtreqless \frac{1}{3}$$

The voting equilibrium is given by

$$\tau^* = \begin{cases} \tau^y(w_m^y(n)) & \text{if} \quad n > 1 \\ \frac{1}{2} & \quad\ \ n < 1 \end{cases}$$

where $w_m^y(n) = \frac{1}{2}\left(1 - \frac{1}{n}\right)$. Substituting this into equation (23), we have

$$\tau^* = \frac{n(1+\rho^y) - \rho^y - \frac{3}{4}\left(1 - \frac{1}{n}\right)^2}{2n(1+\rho^y) - \rho^y - \frac{3}{4}\left(1 - \frac{1}{n}\right)^2} \tag{24}$$

[Figure 7a and 7b are here]

Figure 7a and 7b illustrate the relation between τ^* and n in the intragenerational altruism model. Figure 7a is a case in which the altruism is weak ($\rho^y = 0.01$), and Figure 4b is a strong case ($\rho^y = 0.1$). We can observe that a median voter who belongs to the lowest income group in the younger generation could prefer a lower tax rate if the intragenerational altruism is fairly strong.

[7] See Appendix.

3.5 Distribution

In this subsection we briefly discuss about the distribution of ability. We have qualitatively the same result in more realistic distribution as the uniform one.

First, let us assume a Pareto distribution function such as

$$F(w) = 1 - \left(1 + \frac{w}{b}\right)^{-a} \quad w \in [0, \infty) \tag{25}$$

where $a > 2$ and $b > 0$. The average $\bar{w} = b/(a-1)$ is greater than the median $w_m = b(2^{\frac{1}{a}} - 1)$.

With assumption 3, the average income is given by

$$\bar{y}(\tau) = \frac{2b^2\beta(1-\tau)}{(a-1)(a-2)}$$

which gives $\sigma(\tau) = \tau/(1-\tau)$.

Without altruism, the optimal tax rates are $\tau^o = 0.5$ and

$$\tau^y(w) = \begin{cases} \frac{2nb^2-(a-1)(a-2)w^2}{4nb^2-(a-1)(a-2)w^2} & \text{if} \quad w \leq b\sqrt{\frac{2n}{(a-1)(a-2)}} \\ 0 & \quad\quad w > b\sqrt{\frac{2n}{(a-1)(a-2)}} \end{cases}$$

Solving $1 + nF(w) = \frac{1+n}{2}$, the median voter is given by

$$w_m^y = b\left[\left(\frac{2n}{n+1}\right)^{\frac{1}{a}} - 1\right]$$

The equilibrium tax rate is $\tau^* = \tau^o$ if $n < 1$, and $\tau^* = \tau^y(w_m^y)$ if $n \geq 1$.

[Figure 8 is here]

Figure 8 illustrates the relation between τ^* and n when ability follows the Pareto distribution (We assume $a = 20$).[8] The shape looks like Figure 4, although the quantitative effect of population aging is small because the distribution is skewed to the lower income group.

Next, assume an exponential distribution function such as

$$F(w) = 1 - e^{-dw} \quad w \in [0, \infty) \tag{26}$$

where $d > 0$. The average $\bar{w} = d^{-1}$ is greater than the median $w_m = d^{-1}\ln 2$.

With assumption 3, the average income is given by

$$\bar{y}(\tau) = \frac{2\beta(1-\tau)}{d^2}$$

which gives $\sigma(\tau) = \tau/(1-\tau)$.

[8] The equilibrium tax rate is independent of b. On the one hand, a larger b increases preferred tax rates because the average income increases, which also increases pension benefits. On the other hand, median voters move to the right, which decreases the preferred tax rate. The opposite effects are fully cancelled out in our model.

Without altruism, the optimal tax rate for individuals with ability w in the younger generation is given by

$$\tau^y(w) = \begin{cases} \frac{2n-(dw)^2}{4n-(dw)^2} & \text{if} \quad w \leq \frac{\sqrt{2n}}{d} \\ 0 & \quad\quad w > \frac{\sqrt{2n}}{d} \end{cases}$$

The median voter is given by

$$w_m^y = d^{-1} \ln \frac{2n}{n+1}$$

which gives the equilibrium tax rate when $n \geq 1$ as

$$\tau^* = \tau^y(w_m^y) = \frac{2n - \left(\ln \frac{2n}{n+1}\right)^2}{4n - \left(\ln \frac{2n}{n+1}\right)^2}$$

[Figure 9 is here]

Figure 9 illustrates the relation between τ^* and n when ability follows the exponential distribution.[9] We observe the shape looks like Figure 4 again. They suggests the analysis in a case of uniform distribution would be, at least qualitatively, robust.

4 Concluding remarks

We can observe population aging decreases the size of redistribution especially in a case of the ability-biased intergenerational altruism, and the ability-neutral intergenerational altruism with the median voter belonging to the older generation. This result supports for Razin et-al. (2002). However, we can also observe that the relation is not monotonic but complex (See for example Figure 6 and 7b). Our results suggest 'the public values effect' and 'the social rivalry effect' in Corneo and Grüner (2002) play an important role to answer the question of whether population aging affect the size of the welfare state.

[9] The equlibrium tax rate is independent of d. The reason is the same as Footnote 8.

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Appendix

[Derivation of equation (15)]

From equation (3), we have

$$\frac{\partial U}{\partial \tau} = \frac{\beta}{3}\left[n - 3w^2 - \left(2n - 3w^2\right)\tau\right]$$

First, we know $(\partial U/\partial \tau)_{\tau=1} < 0$. Second, assume that

$$w^2 \leq \frac{n}{3}$$

Then, the optimal tax rate is given by

$$\tau^y(w) = \frac{n - 3w^2}{2n - 3w^2}$$

which is decreasing in w, and increasing in n.

Finally, assume that

$$w^2 > \frac{n}{3}$$

It implies $\partial U/\partial \tau < 0$ for $\forall \tau \in [0, 1]$. Therefore we have $\tau^y(w) = 0$.

[Derivation of equations (17) and (18)]

From (8), we have

$$\frac{\partial W_1^y}{\partial \tau} = \frac{\beta}{3}\left\{n(1+\rho^y) - 3w^2 - [2n(1+\rho^y) - 3w^2]\tau\right\}$$

We know $(\partial W_1^y/\partial \tau)_{\tau=1} < 0$. Assume that $w^2 \leq n(1+\rho^y)/3$. Then we have

$$\tau^y(w) = \frac{n(1+\rho^y) - 3w^2}{2n(1+\rho^y) - 3w^2}$$

which is decreasing in w, and increasing in n and ρ^y. Next, assume that $w^2 > n(1+\rho^y)/3$. Then, we know $\partial W_1^y/\partial \tau < 0$ for any $\tau \in [0, 1]$. It implies $\tau^y(w) = 0$.

From equation (9), we have

$$\frac{\partial W_1^o}{\partial \tau} = \frac{\beta}{3}\left\{n(1+\rho^o) - 3\rho^o w^2 - \left[2n(1+\rho^o) - 3\rho^o w^2\right]\tau\right\}$$

We know $(\partial W_1^o/\partial \tau)_{\tau=1} < 0$. Assume that $w^2 \leq n(1+\rho^o)/(3\rho^o)$. Then we have

$$\tau^o(w) = \frac{n(1+\rho^o) - 3\rho^o w^2}{2n(1+\rho^o) - 3\rho^o w^2}$$

which is decreasing in w and ρ^o, and increasing in n. Next, assume that $w^2 > n(1+\rho^o)/(3\rho^o)$. Then, we know $\partial W_1^o/\partial \tau < 0$ for any $\tau \in [0, 1]$. It implies $\tau^o(w) = 0$.

[Proof of Proposition 1]

The voting equilibrium is given by

$$\begin{aligned} \tau^* &= \tau^y(w_m^y) = \tau^o(w_m^o) \\ n w_m^y + w_m^o &= \frac{1}{2}(1+n) \end{aligned}$$

Assume that

$$\begin{aligned} w_m^y &< \sqrt{\frac{n}{3}(1+\rho^y)} \\ w_m^o &< \sqrt{\frac{n}{3}(1+\frac{1}{\rho^o})} \end{aligned}$$

First, $\tau^y(w_m^y) = \tau^o(w_m^o)$ is equivalent to

$$\frac{(w_m^y)^2}{1+\rho^y} = \frac{\rho^o}{1+\rho^o}(w_m^o)^2$$

which gives

$$w_m^y = \rho w_m^o$$

Substituting this into $nw_m^y + w_m^o = \frac{1}{2}(1+n)$, we have

$$\begin{aligned} w_m^o &= \frac{1+n}{2(1+\rho n)} \\ w_m^y &= \frac{\rho(1+n)}{2(1+\rho n)} \end{aligned}$$

Substituting one of them into $\tau^y(w_m^y)$ or $\tau^o(w_m^o)$, we have

$$\tau^* = \frac{n(1+\rho^y) - \frac{3}{4}\left[\frac{\rho(1+n)}{1+\rho n}\right]^2}{2n(1+\rho^y) - \frac{3}{4}\left[\frac{\rho(1+n)}{1+\rho n}\right]^2}$$

[Proof of Proposition 2]
From equation (18), we know $\tau^* = \tau^o(w_m^o)$ is decreasing in $(w_m^o)^2/n$. Observing w_m^o in Proposition 1, define

$$\phi(n) = \frac{(1+n)^2}{n(1+\rho n)^2}$$

We know the sign of ϕ' is the same as

$$-\rho n^2 + (1-3\rho)n - 1$$

If $0 < \rho < 1/9$, then $\phi' < 0$ when $n < \underline{n}$ or $\overline{n} < n$, and $\phi' > 0$ when $\underline{n} < n < \overline{n}$, where

$$\begin{aligned} \underline{n} &= \frac{1-3\rho-\sqrt{(1-\rho)(1-9\rho)}}{2\rho} \\ \overline{n} &= \frac{1-3\rho+\sqrt{(1-\rho)(1-9\rho)}}{2\rho} \end{aligned}$$

Therefore τ^* is increasing in $n \in (0, \underline{n}), (\overline{n}, \infty)$, and decreasing in $n \in (\underline{n}, \overline{n})$.
If $1/9 \leq \rho < 1$, then $\phi' < 0$ for any $n > 0$. In this case, τ^* increases with n.

[Derivation of equation (23)]

From equation (14), we have

$$\frac{\partial W_3^y}{\partial \tau} = \frac{\beta}{3}\left\{n(1+\rho^y) - \rho^y - 3w^2 - [2n(1+\rho^y) - \rho^y - 3w^2]\tau\right\}$$

We know $(\partial W_3^y/\partial \tau)_{\tau=1} < 0$. Assume that

$$w^2 \leq \frac{n(1+\rho^y) - \rho^y}{3}$$

Then, the optimal tax rate is given by

$$\tau^y(w) = \frac{n(1+\rho^y) - \rho^y - 3w^2}{2n(1+\rho^y) - \rho^y - 3w^2}$$

Next, assume that

$$w^2 > \frac{n(1+\rho^y) - \rho^y}{3}$$

which implies $\partial W_3^y/\partial \tau < 0$ for $\forall \tau \in [0, 1]$. Therefore we have $\tau^y(w) = 0$.

Figure 1. Ability-biased intergenerational altruism

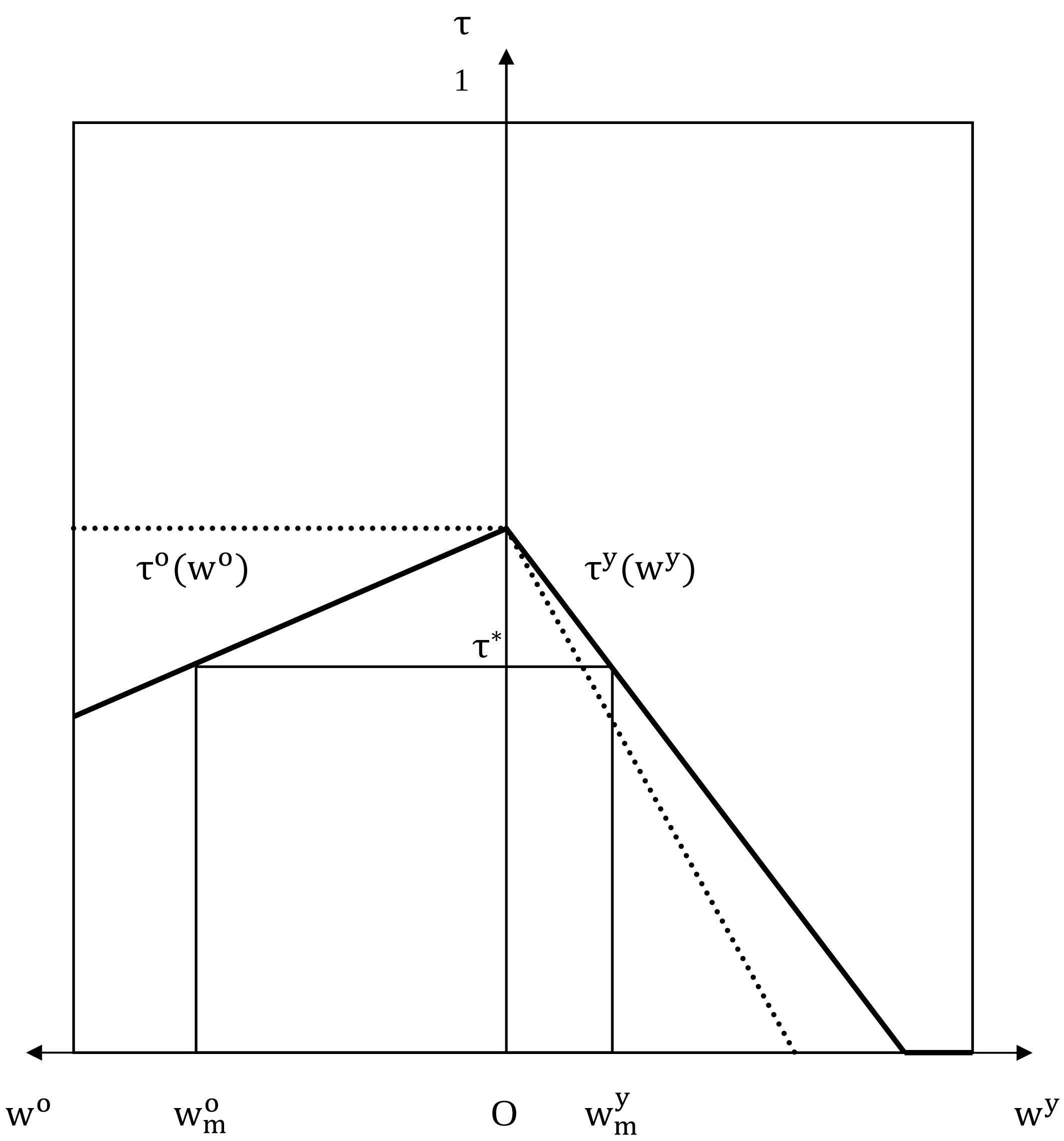

Figure 2. Ability-neutral intergenerational altruism

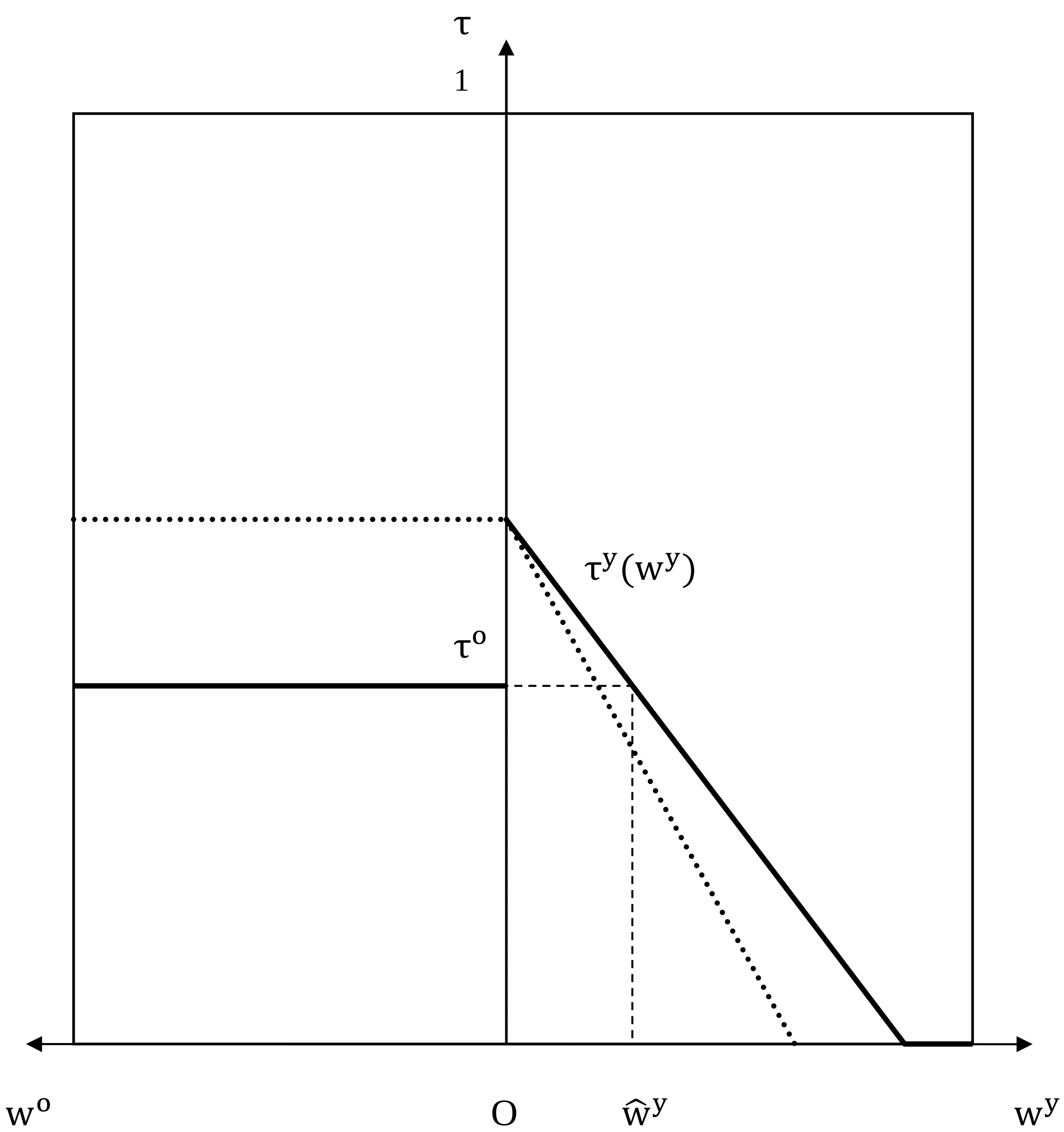

Figure 3. Intragenerational altruism

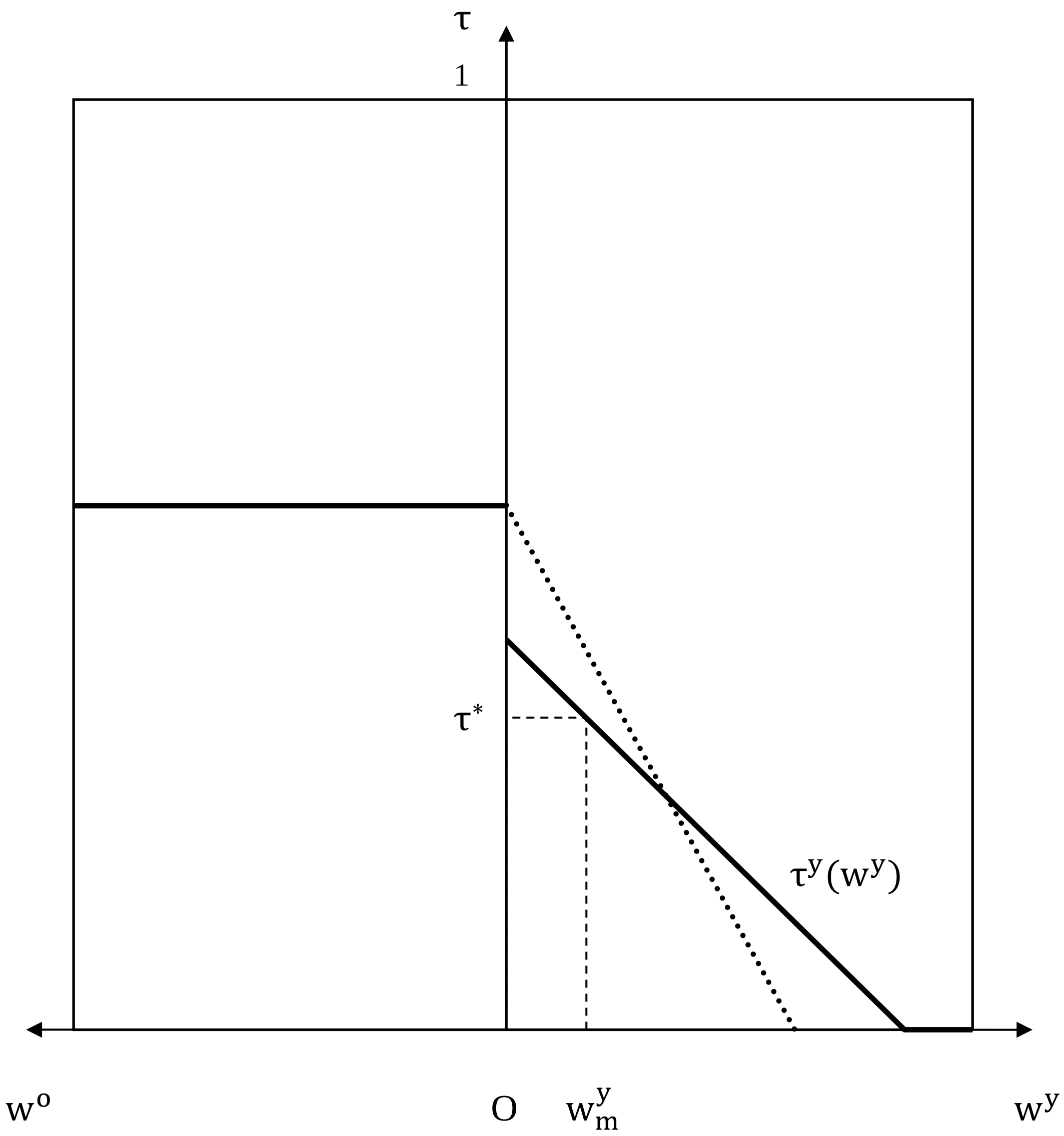

Figure 4. Basic model

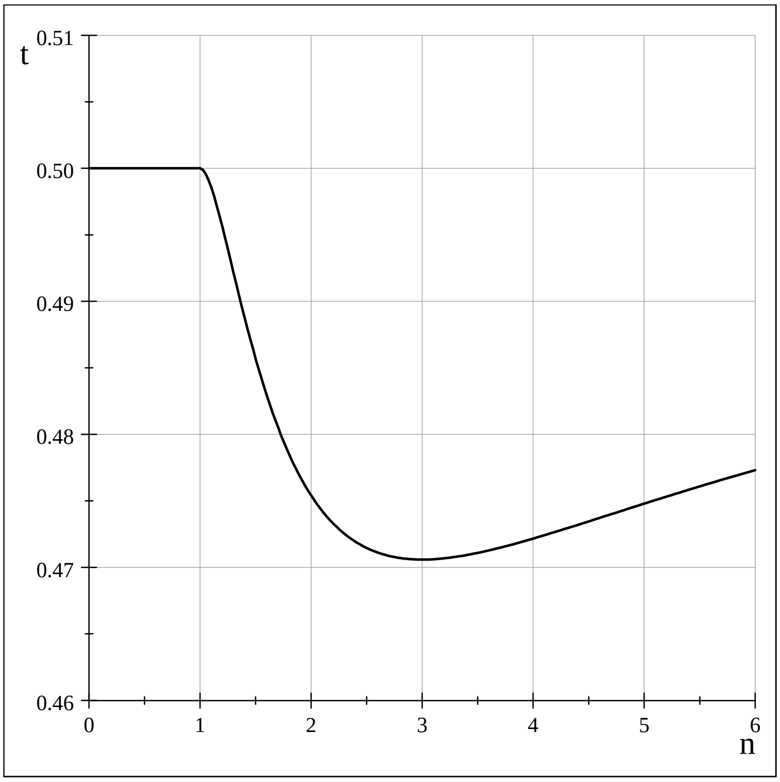

Figure 5a. Ability-biased intergenerational altruism ($\rho^y = 0$, $\rho^o = 0.01$)

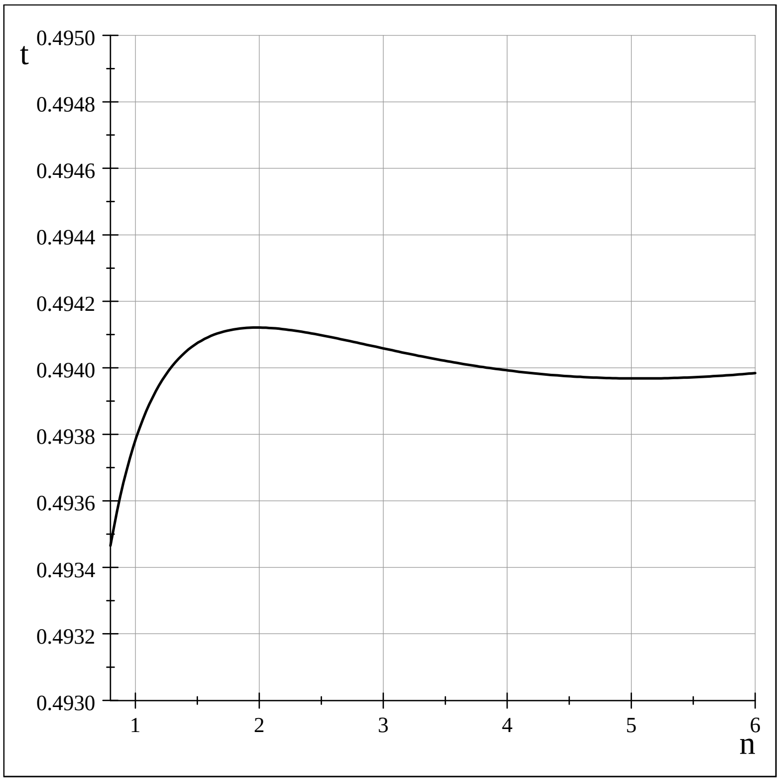

Figure 5b. Ability-biased intergenerational altruism ($\rho^y = \rho^o = 0.1$)

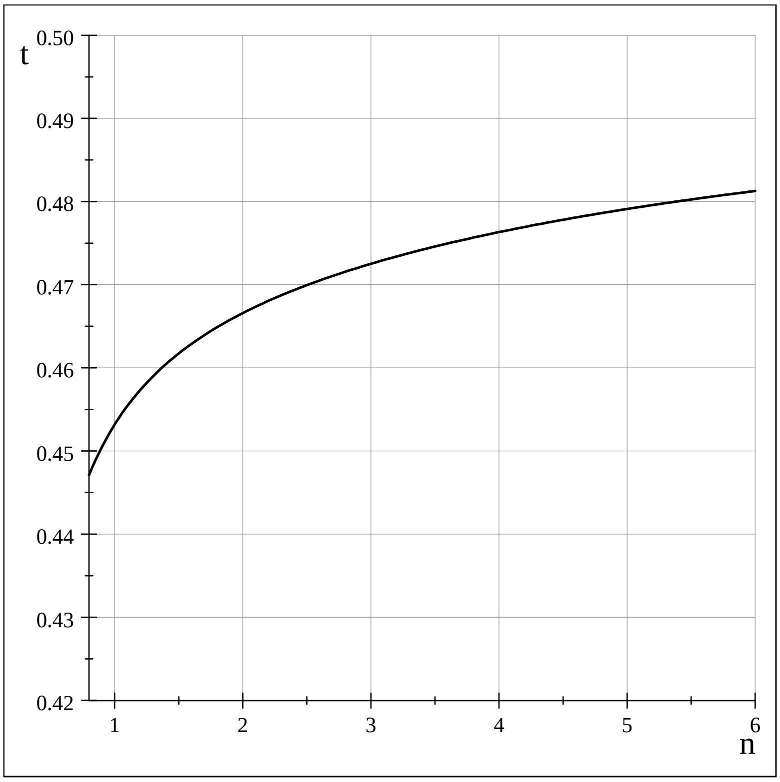

Figure 6. Ability-neutral intergenerational altruism ($\rho^y = \rho^o = 0.1$)

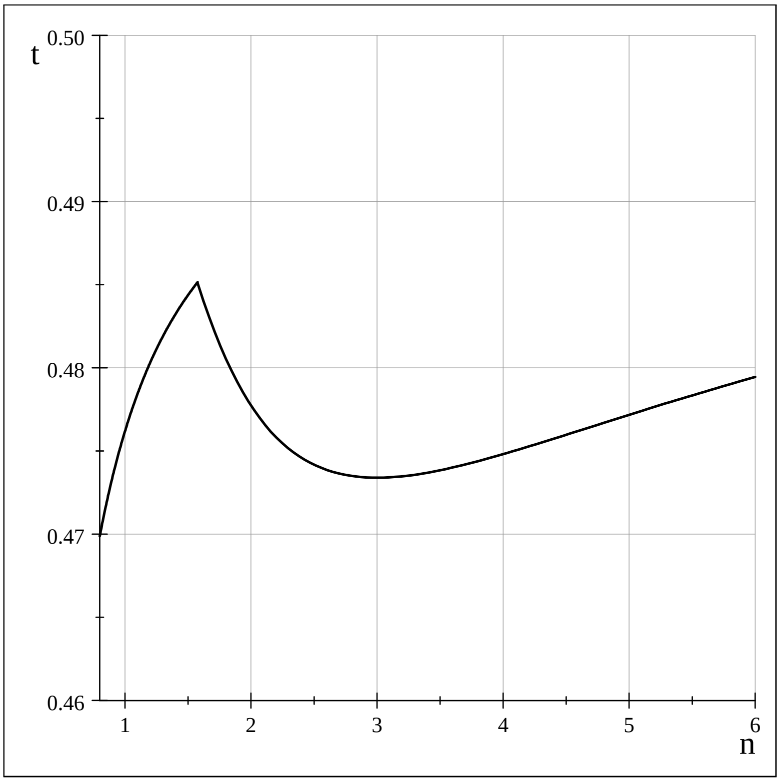

Figure 7a. Intragenerational altruism ($\rho^y = 0.01$, $\rho^o = 0$)

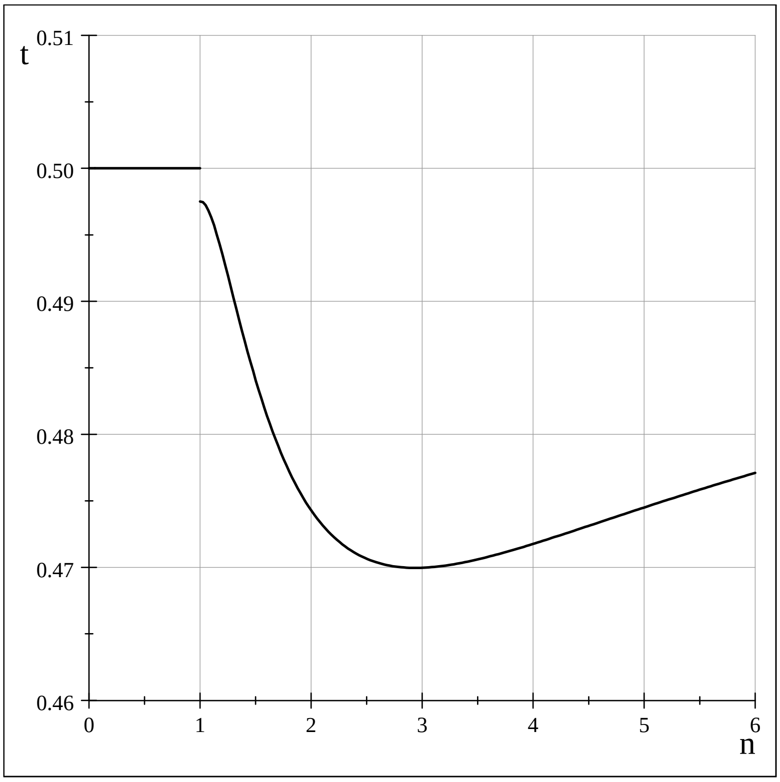

Figure 7b. Intragenerational altruism ($\rho^y = 0.1,\ \rho^o = 0$)

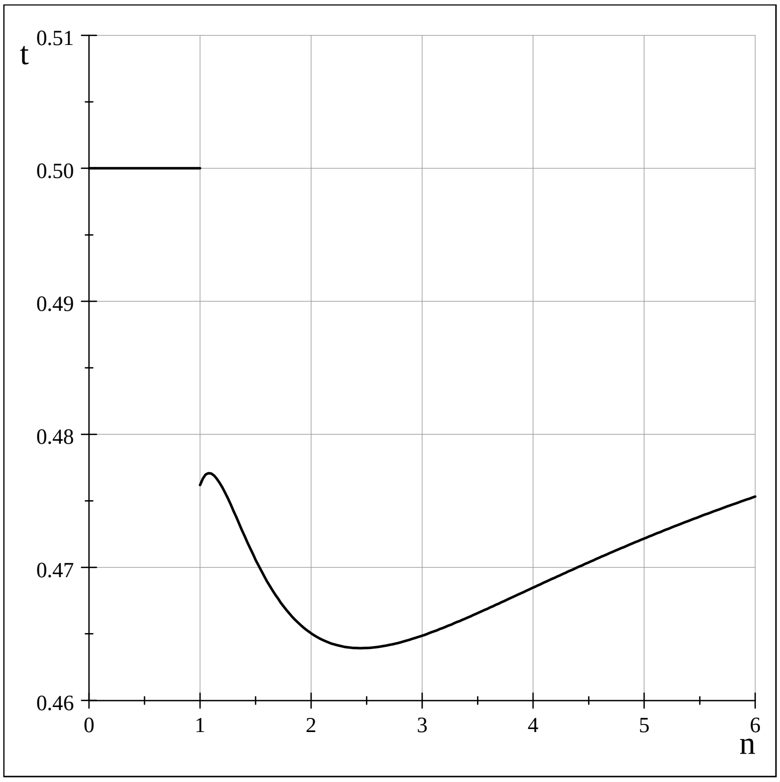

Figure 8. Pareto distribution

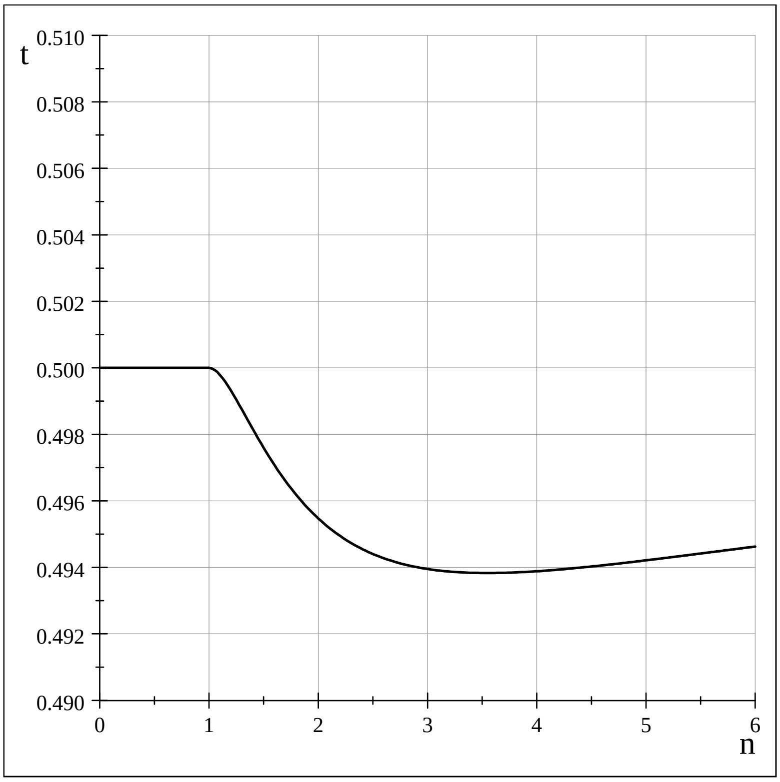

Figure 9. Exponential distribution