

# Workshop on Probability at Kansai University

**Date:** January 31 (Sat), 2015

**Venue:** Kansai University (Senriyama Campus),  
Centenary Memorial Hall, Meeting Room 1

## Program

- 9:00 – 9:35** Jun Masamune (Tohoku University)  
Essential self-adjointness of the Hodge-Laplacian and  $p$ -negligible boundary
- 9:40 – 10:15** Gerald Trutnau (Seoul National University)  
On countably skewed Brownian motion with accumulation point
- 10:20 – 10:55** Minjung Gim (Seoul National University)  
Explicit Criterion for Recurrence of Energy forms perturbed by Divergence free vector fields
- 11:05 – 11:40** Daehong Kim (Kumamoto University)  
General analytic characterization of gaugeability for Feynman-Kac functionals
- 11:45 – 12:20** Panki Kim (Seoul National University)  
Fractional time Stochastic partial differential equations
- 12:20 – 13:40** Lunch
- 13:40 – 14:15** Yusuke Miura (Tohoku University)  
The Girsanov transform of non-local Dirichlet forms
- 14:20 – 14:55** Masatoshi Fukushima (Osaka University)  
Liouville property of harmonic functions of finite energy I
- 15:00 – 15:35** Kazuhiro Kuwae (Kumamoto University)  
Liouville property of harmonic functions of finite energy II
- 15:45 – 16:20** Jose Luis Perez (Universidad Nacional Autonoma de México)  
A Random Matrix Approximation for the Non-commutative Fractional Brownian Motion
- 16:25 – 17:00** Jian Wang (Fujian Normal University/Kyoto University)  
Intrinsic Ultracontractivity for General Lévy processes on Bounded Domains
- 17:05 – 17:40** Liping Li (Fudan University)  
On structure of regular subspaces of 1-dim Brownian motion

**Organizers:** Toshihiro Uemura (Kansai University), Kazutoshi Yamazaki (Kansai University),  
Yuichi Shiozawa (Okayama University)

# Abstracts

## Essential self-adjointness of the Hodge-Laplacian and $p$ -negligible boundary

JUN MASAMUNE (TOHOKU UNIVERSITY)

A fundamental problem in the Hodge-Kodaira theory of an incomplete Riemannian manifold is determining the self-adjoint extensions of the Hodge-Laplacian. In particular, it is striking that the Cauchy boundary of the manifold being polar yields the essential self-adjointness of the scalar-Laplacian, defined on a properly chosen domain; however, not the Hodge-Laplacian as one can see by Cheeger's theory. In this talk, we will discuss a new progress in this problem which was obtained in a collaborative effort with J. Takahashi.

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## On countably skewed Brownian motion with accumulation point

GERALD TRUTNAU (SEOUL NATIONAL UNIVERSITY)

Countably skewed Brownian motion (CSBM) is a special case of distorted Brownian motion in dimension one. Existence and pathwise uniqueness of CSBM was presented by Le Gall in 1984 in an abstract frame for some special cases and then explicitly presented by Takanobu in 1986 assuming a uniform, strictly positive distance between the skew reflection points. In this case CSBM is a semimartingale and conservative, i.e. without explosion in finite time. This is not the case when the sequence of skew reflection points has an accumulation point. In this case we shall discuss conditions for existence, pathwise uniqueness, non-explosion, recurrence and positive recurrence, and conditions for CSBM to be a semimartingale. We shall also consider applications. (This is joint work with Youssef Ouknine (Cadi Ayyad University Marrakech) and Francesco Russo (ENSTA ParisTech)).

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## Explicit Criterion for Recurrence of Energy forms perturbed by Divergence free vector fields

MINJUNG GIM (SEOUL NATIONAL UNIVERSITY)

For a generalized Dirichlet form on  $L^2(U, \mu)$  given as the closed extension of

$$\sum_{i,j=1}^d \int_U a_{ij}(x) \partial_i f(x) \partial_j g(x) \mu(dx) - \sum_{i=1}^d \int_U b_i(x) \partial_i f(x) g(x) \mu(dx)$$

for  $f, g \in C_0^\infty(U)$  where  $B(x) = (b_1(x), \dots, b_d(x)) \in L_{loc}^2(U, R^d, \mu)$  is a divergence free vector field, (ie.  $\sum_{i=1}^d \int_U b_i(x) \partial_i f(x) \mu(dx) = 0$  for all  $f \in C_0^\infty(U)$ ), we present a sufficient conditions on measurable coefficients for recurrence under irreducibility condition. In particular, we find an explicit condition determined by  $\mu$ -volume growth of balls. (This is joint work with Gerald Trutnau (SNU))

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# General analytic characterization of gaugeability for Feynman-Kac functionals

DAEHONG KIM (KUMAMOTO UNIVERSITY)

In this talk, we give a general analytic characterization of the gaugeability for Feynman-Kac functionals involving continuous additive functionals locally of zero energy generalizing the earlier work on the analytic characterization of the gaugeability of Feynman-Kac functional for absorbing Brownian motion on a bounded domain given by Aizenman-Simon (1982). The result also extends the previous known results on the analytic characterization of the gaugeability of generalized Feynman-Kac functionals in the framework of symmetric Markov processes satisfying irreducibility condition and absolute continuity condition.

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# Fractional time Stochastic partial differential equations

PANKI KIM (SEOUL NATIONAL UNIVERSITY)

In this talk, we introduce a class of stochastic partial differential equations (SPDEs) with fractional time-derivatives, and study the  $L_2$ -theory of the equations. This class of SPDEs can be used to describe random effects on transport of particles in medium with thermal memory or particles subject to sticking and trapping.

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# The Girsanov transform of non-local Dirichlet forms

YUSUKE MIURA (TOHOKU UNIVERSITY)

Let  $(\mathcal{E}, \mathcal{D}(\mathcal{E}))$  be a conservative strong local Dirichlet form. It is known that if a function locally in  $\mathcal{D}(\mathcal{E})$  has a finite energy measure, then the Girsanov transformed process generated by it is also conservative and never hits the nodal set of the function. We extend these results to more general Dirichlet forms with non-local parts. For the proof, we use generalized Fukushima's decomposition shown by K. Kuwae and check the condition for the uniform integrability of the exponential martingale introduced by Z.-Q. Chen.

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# Liouville property of harmonic functions of finite energy I

MASATOSHI FUKUSHIMA (OSAKA UNIVERSITY)

When is any harmonic function belonging to the reflected Dirichlet space reduced to a constant function? I give an affirmative answer for energy form on  $\mathbb{R}^n, n \geq 2$ , determined by any positive smooth function depending only on the radial part of the variable. A probabilistic significance of this property will be also discussed.

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# Liouville property of harmonic functions of finite energy II

KAZUHIRO KUWAE (KUMAMOTO UNIVERSITY)

I will talk on a Liouville property of  $\mathcal{E}$ -harmonic function of finite energy in the framework of symmetric Markov processes: if any bounded  $\mathcal{E}$ -harmonic function  $u \in \mathcal{F}_{\text{loc}}$  is

a constant  $\mathcal{E}$ -q.e., then any  $\mathcal{E}$ -harmonic function of finite energy is also a constant  $\mathcal{E}$ -q.e. The result generalizes a result by Atsuji in the framework of diffusion processes on Riemannian manifold.

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## A Random Matrix Approximation for the Non-commutative Fractional Brownian Motion

JOSE LUIS PEREZ (UNIVERSIDAD NACIONAL AUTONOMA DE MÉXICO)

A functional limit theorem for the empirical measure-valued process of eigenvalues of a matrix fractional Brownian motion is obtained. It is shown that the limiting measure-valued process is the non-commutative fractional Brownian motion recently introduced by Nourdin and Taqqu. Young and Skorohod stochastic integral techniques and fractional calculus are the main tools used. Also the case of a Non-commutative Fractional Poisson Process will be discussed, in terms of an approximation based on the fractional Wishart process.

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## Ultracontractivity for General Levy processes on Bounded Domains

JIAN WANG (FUJIAN NORMAL UNIVERSITY/KYOTO UNIVERSITY)

We prove that general (not necessarily symmetric) Lévy process is intrinsically ultracontractive on bounded domain  $D$  without regular condition, provided that the Lévy measure satisfies

$$\nu(B(x, r)) > 0, \quad r > 0$$

for any  $|x| \leq R_0$  and some  $R_0 > 0$ .

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## On structure of regular subspaces of 1-dim Brownian motion

LIPING LI (FUDAN UNIVERSITY)

The main purpose of this paper is to explore the structure of regular subspaces of 1-dim Brownian motion. As outlined in [FMG] every such regular subspace can be characterized by a measure-dense set  $G$ . When  $G$  is open,  $F = G^c$  is the boundary of  $G$  and, before leaving  $G$ , the diffusion associated with the regular subspace is nothing but Brownian motion. Their traces on  $F$  still inherit the inclusion relation, in other words, the trace Dirichlet form of regular subspace on  $F$  is still a regular subspace of trace Dirichlet form of one-dimensional Brownian motion on  $F$ . Moreover we have proved that the trace of Brownian motion on  $F$  may be decomposed into two parts, one is the trace of the regular subspace on  $F$ , which has only the non-local part and the other comes from the orthogonal complement of the regular subspace, which has only the local part. Actually the former one is a non-local Dirichlet form whereas the latter one has non-trivial local part. The remaining information, i.e. the information of strongly local part of trace Dirichlet form of one-dimensional Brownian motion on  $F$ , is contained in the orthogonal complement of regular subspace corresponds to a time-changed Brownian motion after a darning transform.

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