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A Note on Fertility and Revenue Variance

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Abstract

This paper investigates the relationship between fertility and revenue variance. Evidence shows that there is positive correlation between income and fertility. We theoretically study this relationship and explain why. Implication from this result is that it is better to change the social system to have more performance-based pay than to have subsidy for higher birth rate.

1 Introduction

This paper investigates the relationship between fertility and revenue variance. The importance of income variance to fertility is that the variance raises the value of children who would be born in the future. One's variance of revenue increases the expectation of income that one and one's children might get in the future and thus one tries to bear more children. We reach to this result by using both Hamiltonian and Bellman equation as optimal control theory. This paper seems to be the first theoretical study regarding the association between fertility and revenue variance. There exsists econometric evidence that income variance and fertility would have positive relationship (Perotti, 1996). This is because, by his paper, in a society where there is large income distribution, government tries to amend it by fiscal policy and thus, low economic growth, whereas fertility has a large and highly significant negative coefficient in the growth regression. There is also a study that investigates theoretically the relationship between fertility rate and income density. If the population density and thus income density is concentrated around discontinuity of human capital, a negative aggregate relationship is likely to be observed and otherwise it would be positive, where discontinuity of human capital is the disindivibilites of human capital investments such as in fixed length of different opportunities (primary school, secondary school, and etc) (Docquier, 2004). In this study only the density is concerned and not the variance of income distribution. Croix and Doepke (2003) includes not only total fertility but also differential fertility in their model. They assume an unequal initial distribution of human capital and show that endogenous fertility rate and schooling time are functions of it. In our model we show that it is rather uncertainty that influence the long-run dynamic behavior of fertility. It is quite natural to assme that the income distribution is enlarged by each agent's revenue rather than differential wage with no uncertainty about their income in the future. Besides, if their income have no uncertainty in the future and just the disparity between them, income distribution would not be enlarged. Further we later show in Section 2 that dynamically optimal fertility rate is not a function of wage, w or its difference. Barro and Becker (1986), Barro and Becker (1988), and Barro and Becker (1989) use Lagrange multiplier for every period that does not allow for any savings in each period. Dahan and Tsiddon (1998) showed theoretically that the fertility and GDP growth follow an inverted-U shaped relationship. They examine the number of offspring for the unskilled, the skilled number of offspring of skilled parents, and the skilled number of offspring of unskilled parents and show that the fertility of the unskilled is higher than that of the skilled which is also higher than that of the last one. Agents choose the higher level of income incurring the cost to become skilled worker along with the economic growth and thus the fertility rate alters. That is, along with the economic growth and to some GDP, agents have high level of income and thus high fertility but hereafter, the price of bearing a child becomes expensive and consequently it brings low fertility. Morand (1999) also reach a similar conclusion that the fertility and GDP growth follow an inverted-U shaped relationship by considering old-age support for child bearing. All of the theoretical models above and most of the literature in this field are along the lines of Barro and Becker (1986; 1988) and Barro and Becker (1989). We pursue for more complete model with regards to economic transition in capital accumulation through savings, levels of fertility, comsumption, and more than one parents. We apply the model of Barro and Sala-i-Martin (1995).

2 The model

We think of assets per effective labor, k as the cumulation of independent identically normally distributed increments of savings and a function of asset prices. Thus, we can write

$$dk_i = \mu k_i dt + \sigma k_i dB, \tag{1}$$

where B is a standardized Brownian motion (Wiener Process) whose increment dk has zero mean and variance dt. This is because that k depends on savings per effective labor and savings depends on the cost to raise children and consumption, c, and price of financial assets that alter stochastically. Thus, increments of k per time depends on savings per time by the effective labor. Because of (1), a competitive market, open economy, and firms' profit maximization brings,

$$r_i = f'(\bar{k_i}), \quad w_i = [f(\bar{k_i}) - \bar{k_i} f'(\bar{k_i})](1 + g_i)^i \tag{2}$$

where $\bar{k_i}$ is the expected value of $\hat{k_i}$ for generation i and $\hat{k_i} \equiv K_i/[(1+g)^i L_i]$ as in Barro and Becker (1989). Hereafter we omit i and use representative agents and write k instead of k_i.

We assume that

$$U = \int_0^T \frac{e^{-\rho\tau}}{1-\theta}\{[N^{\psi} c(n-d)^{\phi}]^{1-\theta} - 1\} d\tau, \tag{3}$$

where c stands for consumption and its price is normalized to one. The above equation is from Barro and Sala-i-Martin (1995) and notations are same as them. The difference with Barro and Sala-i-Martin is that we suggest that N is a function of n. In Barro and Sala-i-Martin, N is the scale of a family where n is the fertility rate. And also we use a different cost function for the education of children. We take into our consideration of the quality of the child and the cost to raise the child to the quality. We maximize U applying Hamiltonian function. Following Becker and Lewis (1973) we notate π the price of q, q their quality (assumed to be the same for all of the children), $N = N(0)$ and $N(t) = N(0)e^{nt}$, and c quantity of other consumptions, where the price of c is normalized to 1. Substituting these notations into a simple budget constraint as in Becker and Lewis (1973), i.e.,

$$k = K(0) + \int_0^T e^{-R(\tau)}\dot{k}([w + rk - nq\pi - c])Nd\tau \geq 0, \tag{4}$$

where $R(\tau) = \int_0^T r(\tau)d\tau$. Hereafter we assume that $r = \rho$. Thus $e^{-R(t)} = e^{-\rho t}$, and we have, here

$$\dot{k}(\dot{s}) = \dot{k}(w + rk - nq\pi - c). \tag{5}$$

We just assume here that $\dot{k}$ is a function of dk and dk is a function of savings per effective labor s, at a time. The reason of this is that, as in Becker and Barro (1988) and Barro and Becker (1989), we include human capital in k. Because k includes human capital, the transfer of it beyond generations might take more than the integral of dk. Rather it is natural to assume that the transfer of k, human captial, to ones' children take several times. It might depend on abilities of children or how parents teach them. It is thus difficult to define and therefore we do not insist how k would be attained by dk and we just mention that $\dot{k}$ is a function of dk. Because the asset yields no cash flows when the asset is bought by s, the value of holding it is its capital appreciation and there is no immediate payout or dividend from the asset. Thus a no-arbitrage or equilibrium condition brings,

$$r\dot{k}dt = E[dk]. \tag{6}$$

Following the model of Dixit and Pindyck (1994, pp.140-142), we therefore get

$$\dot{k} = Ak^{\beta}, \tag{7}$$

where $\dot{k} = 0$ when $k = 0$,

$$A = \frac{(\beta - 1)^{\beta - 1}}{\beta^{\beta} I^{\beta - 1}}, \tag{8}$$

$$\beta = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{(\frac{\mu}{\sigma^2} - \frac{1}{2})^2 + \frac{2r}{\sigma^2}}, \tag{9}$$

and I is the tangible cost of investment or buying an asset, $\dot{k}$. In deriving equation (7), we use

$$\dot{k}^* = k^* - I \tag{10}$$

and

$$\dot{k}^{*\prime}(\dot{s}) = 1. \tag{11}$$

In equation (10), $\dot{k}^*$ and k^* are the critical value where the net gain from the two equals I. Equation (11) is the smooth pasting condition. Thus, Hamiltonian expression becomes as below.

$$
\begin{aligned}
H &= \frac{e^{-\rho t}}{1-\theta}\{[N^{\psi}c(n-d)^{\phi}]^{1-\theta}\} + \lambda_1 e^{-R(t)}\dot{k}([w + rk - nq\pi - c])N \\
&+ \lambda_2(n-d)N. \qquad (12)
\end{aligned}
$$

The second term in RHS is λ_1 multiplied by differentiated budget constraint, where λ_1 and λ_2 are the shadow prices regarding with the two state variables, k and N, whereas c and n are control variables to maximize U in equation (3). We put $T = \infty$ when solving Hamiltonian.

$$
\begin{aligned}
\frac{\partial H}{\partial c} &= [N^{\psi}c(n-d)^{\phi}]^{-\theta}[N^{\psi}(n-d)^{\phi}] - \lambda_1 \beta A k^{\beta-1} = 0 \\
\lambda_1 \beta A k^{\beta-1} &= \frac{[N^{\psi}c(n-d)^{\phi}]^{1-\theta}}{Nc}. \qquad (13)
\end{aligned}
$$

Because of $\frac{\partial H}{\partial n} = \frac{\partial H}{\partial c} = 0$, and $N(t) = N(0)e^{nt}$

$$
\begin{aligned}
\frac{\partial H}{\partial n} &= e^{-\rho t}[N^{\psi}c(n-d)^{\phi}]^{-\theta}[N^{\psi}c\phi(n-d)^{\phi-1}] \\
&+ e^{-\rho t}[N^{\psi}c(n-d)^{\phi}]^{-\theta}[\psi N^{\psi-1}tNc(n-d)^{\phi} \\
&- \lambda_1[e^{-R(t)}(\beta A k^{\beta-1}q\pi N + nq\pi tN)] \\
&+ \lambda_2[N + (n-d)tN] = 0, \qquad (14)
\end{aligned}
$$

where combining the first and second term on the right hand side we get,

$$
e^{-\rho t}[N^{\psi}c(n-d)^{\phi}]^{1-\theta}[\frac{\phi}{n-d} + t\psi] = e^{-\rho t}\lambda_1 \beta A k^{\beta-1} Nc[\frac{\phi}{n-d} + t\psi] \qquad (15)
$$

from equation (13). Thus, $\frac{\partial H}{\partial n}$ becomes

$$\begin{aligned}
\frac{\partial H}{\partial n} &= e^{-\rho(t)}\lambda_1\beta Ak^{\beta-1}Nc[\frac{\phi}{n-d}+t\psi] \\
&- \lambda_1[e^{-R(t)}(\beta Ak^{\beta-1}q\pi N + nq\pi tN)] \\
&+ \lambda_2[N+(n-d)tN]=0. \qquad (16)
\end{aligned}$$

Because of transversality condition, when $t=\infty$, $\lambda_1 w = \lambda_2 N = 0$. Therefore the above equation becomes,

$$\begin{aligned}
\lambda_1\beta Ak^{\beta-1}Nc[\frac{\phi}{n-d}+t\psi] &= \lambda_1\beta Ak^{\beta-1}q\pi N + \lambda_1 nq\pi tN] \\
c[\frac{\phi}{n-d}+t\psi] &= \frac{nq\pi t}{\beta Ak^{\beta-1}}+q\pi \\
c\phi + c\psi t(n-d) &= (n-d)\frac{nq\pi t}{\beta Ak^{\beta-1}}+q\pi \qquad (17)
\end{aligned}$$

and deviding both sides by t and substituting $t=\infty$ we have,

$$\begin{aligned}
c\psi(n-d) &= (n-d)\frac{nq\pi}{\beta Ak^{\beta-1}} \\
n^* &= \frac{c\psi}{q\pi}\beta Ak^{\beta-1}. \qquad (18)
\end{aligned}$$

This is the steady-state dynamically optimal fertility rate with uncertainty regarding the return of human capital examined. We see here in equation (18) that n^* is not a function of w or its difference, because even though we have included wage differences, it would be deleted by transversality condition, $\lambda_1 w = \lambda_2 N = 0$. This result has implication that although revenue difference has no effect on fertility, one's future income variance has large and positive effect on fertitlity. That is, richness *per se* is not encouraged for fertility but, efficiency wage is encouraged, because this expands the uncertainty about one's future income and thus its expectation. The policy implication from this result is that, though there is some direct policy that has some impact on fertility, it might be better to change the social system to have higher birth rate. We see how it is encouraged in Section 3. The expectation of higher return in the future expressed by larger σ lead parents to have more children. Besides, to have those high return with large σ, i.e. uncertainty, associated with their children, one needs numerous children. Because k involves human capital also, expectation towards their own children becomes incentive to give birth to more children. This implication could also be applied to those who get tax exemption. The fact that tax expemption and subsidy to child in poor families have positive effect on fertility could be understood as the future income that the children in the future bring to their family.[1] But for those who are not poor the credit provided and expanded by the government such as the earned income tax credit (ETIC) only produced extremely small reductions in higher order fertility among white women (Baughman and Dickert-Conlin, 2009). The reason of this, understood from our study, is that because ETIC is phased out with additional income above a certain amount over the phase-out range, it only made the uncertainty or variance of revenue including the credit and income smaller. And thus, the value of children in the future are made lower and therefore reduced the fertility.

This endogenous fertility decisions involve some exogenous factors involving the transfer of assets including human capital. Equation (2) is given by the mean of k and each agent's wage and therefore it is endogenous, but because k is the cumulation of independent identically normally distributed increments of savings, dynamically optimal fertility rate involves uncertainty regarding human capital transfer.

[1]See Whittington (1992) and Whittington, *et.al* (1990).

3 Simulation

We can understand from equation (18) that c and n^* have positive and linear relationship. We simulate connection between σ and n^* beneath. In Figure 1, we substitute $c = 10$, $q = 10$, $\pi = 2$, $\psi = 0.2$, $r = 0.05$, $\mu = 0.01$, $k = 5$, and $I = 10$.

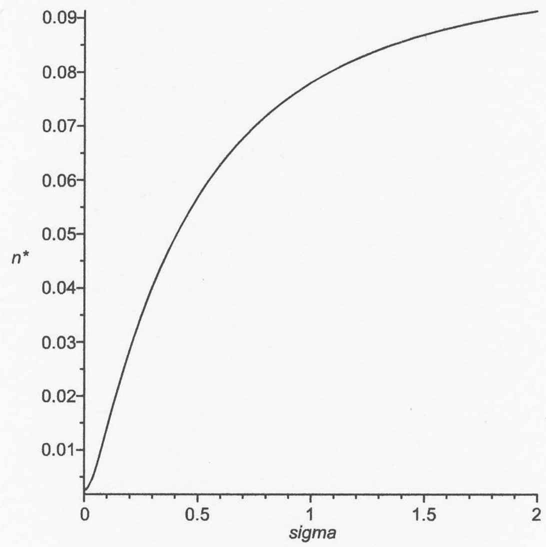

Figure 1: Relationship between optimal fertility and σ.

It is evident that the variance in agents' future income have large effect on fertility. This is because uncertainty regarding human capital transfer towards their children enhance the value of their offspring.

4 Concluding remarks

We study in this paper the positive correlation between revenue variance and fertility. It is the expectation regarding their future income that raises the value of their offspring and thus it becomes the incentive to have more children.

The expectation that parents and their own children might yield higher return in the future lead parents to bear more children. Subsidy is only effective to poor people that could be seen from econometric evidence and the reason that we understand from our study is that subsidy raises the expectation that they might be rewarded by their coming children, but for those who are not poor the revenue variance caused by the subsidy is not large as much as for those who are poor and thus also the value of their offspring. The policy implication from this paper is that it is better to change the social system so that performance-based wage is more accepted.

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