Progressive Taxation, Income Convergence and Economic Growth

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Abstract

The relationship between income inequality and economic growth is one of the controversial issues in macroeconomics. In this paper, we investigate a dynamic relationship between income inequality and economic growth in a simple model of endogenous growth with heterogeneous households, progressive taxation, and a productive public spending.

We show that the relationship between inequality and growth depends on the initial distribution of income. Starting from a Marxian economy, the relationship is negative. On the other hand, starting from a fundamentally unequal economy, the relationship is at first negative and then positive in the process of income convergence. The welfare effect of progressive taxation and the implication to the growth regression puzzle are also discussed.

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1 Introduction

Economic growth and income inequality have attracted considerable attention in economics because the former is associated with efficiency consideration and the latter with equity consideration. Among the widespread related research, one of the main topics especially in macroeconomics is a dynamic relationship between economic growth and inequality.\(^1\)

With respect to the source of inequality, many papers attribute to capital market imperfection. Special attention has been paid to a borrowing constraint on education loans since Galor and Zeira (1993) demonstrate the important role of the initial distribution of wealth in macroeconomics (Galor and Zang (1997), Owen and Weil (1998), Dahan and Tsiddon (1998), Hazan and Berdugo (2002), and Checchi and García-Peñalosa (2004) among others). A common characteristics of the literature is that a higher income or growth equilibrium would be realized if the initial distribution of income is relatively equal. Thus, we may observe a negative relationship between inequality and growth. Further, suppose that the degree of capital market imperfection is related to a higher market interest rate. Then, we may also observe a positive correlation between inequality and the interest rate.

With this thought experiment, we present an alternative model to explain a non-monotonic relationship between growth and inequality that many developed countries have experienced. One of the critical differences from the literature is to focus on the role of progressive taxation in the positive correlation between income inequality and the interest rate.\(^2\) Our new point of view is that the causality between the interest rate and income inequality may be inverse. Suppose that a public spending is financed by a progressive income tax, and that the public spending is productive like Barro (1990). When the tax scheme is progressive, income inequality would increase tax revenues given that the average income is constant. An increase in the public spending enhances marginal product of private capital because the public spending is productive. Therefore we can observe a positive relationship between income inequality and the interest rate. Another role of progressive taxation is of course a fiscal device to achieve equal society.\(^3\) With respect to the source of income inequality, we use two assumptions. First, the initial wealth difference which is assumed to come from firm ownership is inherited from generation to generation. Second, the initial allocation of capital among households is historically determined. In this setting, we examine the law of motion of income distribution as well as a dynamic relationship between inequality and growth.

Our model is related to Bovenberg and van Ewijk (1997), Yamarik (2001), and Li and Sarte (2004). Bovenberg and van Ewijk (1997) examine the trade-off between equity and efficiency in a model of endogenous growth with overlap-

\(^1\) See Bénabou (1996) for an elaborate review of the Kuznets (1955) hypothesis.

\(^2\) Tax progressivity has been examined mainly in a context of “the principle of equal sacrifice” (Young (1988, 1990), Berliant and Gouveia (1993), Mitra and Ok (1996, 1997), Neill (2000)), or a stabilizing device (Guo and Lansing (1998), Guo (1999), Slobodyan (2005). Using a progressive taxation, Sarte (1997), Sorger (2002), and Li and Sarte (2004) show the existence of a non-degenerate distribution of income when individuals are heterogenous in their time preference.

\(^3\) In a computable general-equilibrium framework, Altig and Carlstrom (1999) attributes the increased income inequality throughout the 1980’s in the United States to the marginal tax rate changes instituted by the Tax Reform Act 1986 (TRA86).
ping generations. Their model is useful to examine intra- and intergenerational equity, but it ignores the changes in the interest rate by assuming a small open economy. Introducing a nonlinear tax structure into an AK growth model, Yamakim (2001) shows that a more progressive tax scheme lowers the transitional growth rate and raises the speed of convergence. The dynamics is similar to ours, but it ignores the distributional aspect by assuming representative agents. Li and Sarte (2004) introduces a progressive tax scheme into Barro (1990) model, showing that sustainable income inequality is feasible even if households have heterogeneous discount rates, and that the decrease in progressivity associated with the Tax Reform Act 1986 had a significant effect on the US income inequality. Our model is different from Li and Sarte (2004) in that we assume heterogeneity arises from wealth inequality and that we examine the transitional growth rate as well as income inequality.

We have four main results. First, there exist two equilibria in the long run. One is an equal economy in which each household has the same amount of income although the income components are different. Second is a Marxian economy in a sense that someone becomes a capitalist who possesses all capital over time and devotes himself to capital investment, and that others become workers in a worker-owned firm who consume all income and do not have any capital at any point in time. We show that the equal economy is stable and the Marxian is unstable. This result suggests that the outcomes obtained in a two-class economy model should be treated carefully if it is constructed in a static framework.

Second, the relationship between inequality and growth in the transition process is not monotonic. It is shown that, starting from the neighborhood of the Marxian economy, we can observe a negative relationship between inequality and growth, which is consistent with the empirical evidence in Perrson and Tabellini (1994). This is not true, however, if the initial economy is fundamentally unequal, which means rich people possess both capital and firm ownership and poor people do not. We show that, starting from the fundamentally unequal economy, the relationship is at first positive and then negative. The inverse U shaped pattern supports the so-called Kuznets hypothesis. The non-monotonic pattern comes from the distributional changes in capital allocation. In a fundamentally unequal economy, poor people increase capital investment sharply because they face a lower marginal tax rate. On the other hand, rich people decrease capital investment not only because they face a higher marginal tax rate but also because they can afford consumption from another source of income, that is, firm ownership. When the income inequality is fairly large, the rapid growth of capital of the poor pushes up the growth of aggregate capital. However, it turns to decline in the process of income convergence because the capital share of the rich is nevertheless dominant. Our compound results could explain why some researchers support the Kuznets hypothesis and others do not.

Third, we show that the welfare effect of progressive taxation is strictly negative for all households once the economy converges to the equal one. This implies that the government should abandon tax progressivity once the income inequality becomes admissible. The optimal tax progressivity in the transition

\footnote{In an endogenous growth model with accidental bequests, Miyazawa (2006) shows the similar result can be obtained in a context of population aging.}
process would be given by the magnitude of the welfare gain from the speed of convergence relative to the welfare loss of progressive taxation associated with the distortionary effect on capital investment.

Finally, our model could answer the question of why some researchers estimate a positive growth effect of public spending and others do not. We find that an increase in the tax rate facing a household who earns the average income has a various impact on economic growth depending on the distribution of income. For example, an increase in the tax rate enhances economic growth if the income distribution is fairly equal, while it hampers growth if the income inequality is large. This demonstrates that the distributional consideration is relevant to a controversial issue of growth regression.

The construction of the paper is as follows. We setup the basic model in Section 2. A positive relationship between income inequality and the interest rate is derived. In Section 3, we examines the law of motion of income distribution in a two class economy. There may be multiple equilibria, but owing to tax progressivity, the stable long-run equilibrium is uniquely determined. In Section 4, we examine the dynamic relationship between inequality and growth. In Section 5, we discuss the welfare effect of progressive taxation and the implications of our model to the growth regression. Final section concludes the paper.

2 Basic model

First let us set up a tax scheme such as

\[ \frac{z_i}{\bar{z}} = \left( \frac{y_i}{\bar{y}} \right)^{1-\phi} \quad (0 \leq \phi \leq 1) \]  

(1)

where \(y_i\) and \(z_i\) respectively stand for before- and after-tax income of household \(i \in I\). An upper bar indicates the corresponding average income. Equation (1) implies that the income status of household \(i\) relative to the average shrinks but the order is preserved under the tax scheme. Assuming that each household takes the average income as given, we can avoid the incentive compatibility problem such as Berliant and Gouveia (1993). If \(\phi = 0\), then the scheme corresponds to a proportional one. In another extreme case that \(\phi = 1\), everyone has the same amount of after-tax income. Thus, a larger value of the parameter \(\phi\) corresponds to a higher degree of tax progressivity. Obviously, the tax scheme satisfies the Lorenz dominance if \(\phi > 0\).

The scheme (1) is fairly common in the related literature. Let us assume that a person who earns the average income faces a proportional tax rate of \(1 - \eta\) (\(0 < \eta < 1\)), that is,

\[ \bar{z} = \eta\bar{y} \]  

(2)

Substituting equation (2) into equation (1), we have the average tax rate such as

\[ \tau_i = 1 - \eta x_i^{-\phi} \]  

(3)

where \(x_i\) stands for the income status relative to the average,

\[ x_i = \frac{y_i}{\bar{y}} \]  

(4)
Equation (3) is the same formula as Guo and Lansing (1998), Guo (1999), and Yamarik (2001). Further, since the marginal tax rate is given by $\tau_i^m = 1 - (1 - \phi)\eta x_i^{-\phi}$, we have

$$\frac{1 - \tau_i^m}{1 - \tau_i} = 1 - \phi$$

which is the same as equation (6) in Bovenberg and van Ewijk (1997).

A household $i \in I$ maximizes

$$\int_0^\infty e^{-\rho t} \ln c_i(t) dt$$

subject to the budget constraint

$$(1 - \tau_i)y_i = c_i + k_i$$

where $c_i > 0$ and $k_i \geq 0$ stand for consumption and capital, respectively. $\rho > 0$ is a rate of time preference.

Households are heterogenous with respect to their inherited wealth. Specifically, we assume that wealth differences come from firm ownership, that is, the share of profit. Denoting the profit share of household $i$ by $h_i$ ($0 \leq h_i \leq 1$), the household income is given by $y_i = r k_i + \pi h_i$ where $r$ and $\pi$ stand for a rental price of capital and a profit, respectively.

The optimality condition requires

$$\frac{\hat{c}_i}{c_i} = (1 - \phi)\eta x_i^{-\phi} r - \rho$$

Equation (7) implies that the consumption growth for a higher income status household is smaller because it faces a higher marginal tax rate.

The production function is the same as Barro (1990):

$$y = Ak^\alpha g^{1-\alpha}$$

where $y, k, g$ stand for output, private capital in a broad sense, and a public spending, respectively. $A > 0$ is total factor productivity and $0 < \alpha < 1$ is the income share of private capital.

Assuming that the capital market is perfectly competitive, we have

$$r = \alpha A \left(\frac{g}{k}\right)^{1-\alpha}$$

$$\pi = (1 - \alpha)Ak^\alpha g^{1-\alpha}$$

The capital market clears when

$$k = \sum_{i=1}^I k_i$$

The government budget constraint is given by

$$\sum_{i=1}^I \tau_i y_i = g$$
The resource constraint,
\[ y = \sum_{i=1}^{I} c_i + g + \dot{k} \]  \hspace{1cm} (13)

can be derived from the fact that households are the only residual claimants,
\[ \sum_{i=1}^{I} h_i = 1 \]  \hspace{1cm} (14)

Substituting equations (3) and (4) into equation (12), and using \( y = y/I \) and equations (8) and (9), we have the rental price of capital as a function of the distribution of income,
\[ r = r(x) = \alpha A^{\frac{1}{I}} \left( 1 - \eta I^{-1} \sum_{i=1}^{I} x_i^{1-\phi} \right)^{\frac{1-\alpha}{\eta}} \]  \hspace{1cm} (15)

where \( x = (x_1, \ldots, x_I) \) stands for income distribution and \( \sum_{i=1}^{I} x_i = I \). Since a manifold \( \bar{x} = \sum_{i=1}^{I} x_i^{1-\phi} \) is convex with respect to the origin, the corresponding contour on a plane \( \sum_{i=1}^{I} x_i = I \) is figured as a circle, and the vertex lies at \((1, \ldots, 1)\). Thus, equation (15) implies that the rental price of capital is lower when the distribution of income is more equal, given that the tax scheme is progressive.\(^5\) Under a progressive tax scheme, tax revenues increase with income dispersion given that the average income is constant. Thus, income equality decreases the public spending, which lowers the marginal product of capital. Our model contrasts with Bovenberg and van Ewijk (1997) and Yamarik (2001) in that the rental price of capital depends on the distribution of income. Figure 1 illustrates a case of \( I = 2 \).\(^6\)

[Figure 1 is here]

### 3 Dynamics of income distribution

In this section, we examine the dynamics of income distribution under the progressive tax scheme. To focus on the main feature of the model economy, we assume a two class economy such as

\[ (h_1, h_2) = (0, 1) \]  \hspace{1cm} (16)

Equation (16) implies that household 1 earns capital income \( r_k_1 \) only, and household 2 earns capital income \( r_k_2 \) as well as unearned income \( \pi \).\(^7\) Appendix 5If the tax scheme is a proportional one \( (\phi = 0) \), then the rental price of capital is independent of the distribution of income and has a constant value of \( r = \alpha A^{\frac{1}{I}} (1 - \eta)^{\frac{1-\alpha}{\eta}} \), which corresponds to the original model of Barro (1990).

5Assumed parameters are \( \alpha = 0.8, \eta = 0.8, \phi = 0.1, \rho = 0.04, \) and \( A = 0.25 \).

7In this two class economy, the Gini coefficient is simply given by \( |y_1 - y_2|/(2y) = |x_1 - 1| \).
shows that the main feature of our model is preserved in a more general setting. From equation (15), the rental price of capital is given by

\[ r = r(x_1, x_2) = \alpha A^{\frac{1}{\alpha}} \left[ 1 - \frac{\eta}{2} \left( x_1^{1-\phi} + x_2^{1-\phi} \right) \right]^{\frac{1-\alpha}{\alpha}} \]  

(17)

where \( x_1 + x_2 = 2 \).

In addition to wealth inequality expressed by equation (16), the initial allocation of capital also matters for the dynamics of income distribution. To see this, substituting equations (8) and (9) into equation (4), the distribution of income \( (x_1, x_2) \) is given by

\[ x_1 = 2 \alpha \frac{k_1}{k} \]  

(18.1)

\[ x_2 = 2 - x_1 \]  

(18.2)

where \( k = k_1 + k_2 \). Nonnegative conditions require \( 0 \leq x_1 \leq 2\alpha \). Equations (18.1) and (18.2) imply that the allocation of capital determines the distribution of income in each point in time. Since capital is a state variable, we are able to take the income distribution as a state variable in the following analysis.

It is convenient to derive at first the law of motion of the consumption-capital ratio, which is defined by \( \omega_i = c_i/k_i \), from equations (6) and (7). For household 1 who earns \( y_1 = r k_1 \), we have

\[ \frac{\dot{c}_1}{c_1} = (1-\phi)\eta x_1^{-\phi} r - \rho \]  

\[ \frac{\dot{k}_1}{k_1} = \eta x_1^{-\phi} r - \frac{c_1}{k_1} \]

where \( r \) is given by equation (17). Differentiation gives

\[ \frac{\dot{\omega}_1}{\omega_1} = \omega_1 - \rho - \phi \eta x_1^{-\phi} r \]  

(19)

In the same way, for household 2 who earns \( y_2 = r k_2 + \pi \),

\[ \frac{\dot{\omega}_2}{\omega_2} = \omega_2 - \rho - \eta x_2^{-\phi} \left( \phi r + \frac{\pi}{k_2} \right) \]

Substituting equations (9), (10), (18.1), and (18.2) into this, we have

\[ \frac{\dot{\omega}_2}{\omega_2} = \omega_2 - \rho - \eta x_2^{-\phi} \left[ \phi r + \frac{2(1-\alpha)}{x_2 - 2(1-\alpha)} \right] \]  

(20)

Equations (19) and (20) imply that the law of motion of the consumption-capital ratio is regulated by the distribution of income \( (x_1, x_2) \).

The next step is to analyze how the income distribution \( (x_1, x_2) \) evolves over time. From equation (17.1), the law of motion of income status \( x_1 \) can be captured by the difference between the growth rate of \( k_1 \) and \( k \):

\[ \frac{\dot{x}_1}{x_1} = \frac{\dot{k}_1}{k_1} - \frac{k}{k} \]
Let us notice that the growth rate of aggregate capital can be decomposed as

\[ \dot{k} = \frac{k_1}{k} \dot{k}_1 + \frac{k_2}{k} \dot{k}_2 \]

Substituting this into the equation above, we have

\[ \frac{\dot{x}_1}{x_1} = \frac{k_2}{k} \left( \frac{\dot{k}_1}{k_1} - \frac{\dot{k}_2}{k_2} \right) \]

Finally, substituting equations (7), (9), (10), and (18.1) into this, we have

\[ \frac{\dot{x}_1}{x_1} = (1 - \frac{x_1}{2\alpha}) \left[ \eta r \left( x_1^{-\phi} - x_2^{-\phi} \right) + \omega_2 - \omega_1 \right] - \frac{1 - \alpha}{\alpha} \eta r x_2^{-\phi} \]  

(21)

The dynamics of \((x_1, x_2, \omega_1, \omega_2, r)\) is characterized by equations (17), (19), (20), (21), and \(x_1 + x_2 = 2\).

It would be insightful to illustrate a phase diagram of \((\omega_1, \omega_2)\) on a plane \(x_1 = x\) \((0 < x < 2\alpha)\) because, given that \(x_1 = x\), we have \(x_2\) and \(r\) immediately.

[Figure 2 and 3 are here]

Figure 2 shows that the pair of consumption-capital ratios \((\omega_1, \omega_2)\) is immediately adjusted to \(E(\omega_1(x), \omega_2(x))\), where \(\omega_i(x)\) is given by setting \(\dot{\omega}_i = 0\) in equations (19) and (20), respectively:

\[ \dot{\omega}_1(x) = \rho + \phi \eta x^{-\phi} r \]  

(22.1)

\[ \dot{\omega}_2(x) = \rho + \eta (2 - x)^{-\phi} r \left[ \phi + \frac{2(1 - \alpha)}{2\alpha - x} \right] \]  

(22.2)

Since the equilibrium point \(E\) moves according to \(x\), we are able to illustrate the saddle path in a space of \((\omega_1, \omega_2, x_1)\) (See Figure 3). As \(x\) goes to the lower bound, \(\dot{\omega}_1(x)\) goes to infinity and \(\dot{\omega}_2(x)\) converges to \(\dot{\omega}_2(0)\). As \(x\) becomes larger, \(\dot{\omega}_1(x)\) decreases while \(\dot{\omega}_2(x)\) decreases at first and then increases. As \(x\) goes to the upper bound, \(\dot{\omega}_1(x)\) converges to \(\dot{\omega}_1(2\alpha)\) and \(\dot{\omega}_2(x)\) goes to infinity. Thus, the saddle path comes from south and goes to east from a bird’s eye view.

We are able to see in which direction \(x_1\) moves at \(x_1 = x\) by substituting equations (22.1) and (22.2) into equation (21):

\[ \frac{\dot{x}_1}{x_1} = \eta (1 - \phi) r \left( x_1^{-\phi} - x_2^{-\phi} \right) \left( 1 - \frac{x_1}{2\alpha} \right) \]  

(23)

Equation (23) implies that \(\dot{x}_1 = 0\) has two solutions \(x_1^* = 1\) and \(2\alpha\). The solution \(x_1^* = 1\) implies that both household 1 and 2 have positive capital and that they have the same amount of income in the long run. The solution \(x_1^* = 2\alpha\) implies that household 1 possesses all capital, that is, \(k_2 = 0\). Under a plausible assumption that the income share of private capital is greater than a half, \(\alpha > 1/2\), we know that \(x_1^* = 1\) is globally stable and \(x_1^* = 2\alpha\) is unstable. Figure 4 illustrates the right-hand side of equation (23) as a function of \(x_1\).
The dynamics of income distribution depends crucially on the initial allocation of capital. First, suppose that household 1 possesses all capital and that the firm is instituted by workers \((y_1 = rk, y_2 = \pi)\). This economy corresponds to a Marxian economy in a sense that household 1 becomes a capitalist who possesses all capital over time and devotes himself to capital investment, and that household 2 devotes himself to operating the worker-owned firm who consumes all income and does not have any capital at any point in time. As shown above, this Marxian economy can be an equilibrium but unstable under the progressive tax scheme.

Second, suppose that the initial share of capital for household 1 is negligible \((k_1 \approx 0)\). This fundamentally unequal economy is not sustainable not only because progressive taxation reallocates capital from household 2 to household 1 but also because the rental price of capital decreases with changes in income distribution.

In the long-run, the income distribution converges to \((x_1, x_2) = (1, 1)\). The rental price of capital has the minimum value of \(A_1(1 - \eta)^{1/\alpha - 1}\). From equations (7), (18.1), (18.2), (22.1), and (22.2), the long-run equilibrium is characterized by the following four equations:

\[
\begin{align*}
\frac{\dot{c}_1}{c_1} &= (1 - \phi)\alpha A_1^{1/\alpha} \eta (1 - \eta)^{1/\alpha - 1} - \rho \\
\frac{c_1}{k_1} &= \rho + \phi \alpha A_1^{1/\alpha} \eta (1 - \eta)^{1/\alpha - 1} \\
\frac{c_2}{k_2} &= \rho + \left(\phi + \frac{2(1 - \alpha)}{2\alpha - 1}\right) \alpha A_1^{1/\alpha} \eta (1 - \eta)^{1/\alpha - 1} \\
\frac{k_2}{k_1} &= 2\alpha - 1
\end{align*}
\]

The long-run equilibrium is similar to Barro (1990) and Futagami, Morita, and Shibata (1993) even under a progressive tax scheme. From equations (24), (25), and (26), the growth rate and the consumption share are both maximized when \(\eta = \alpha\). It implies that, given the tax progressivity \(\phi > 0\), the tax rate \((1 - \eta)\) should be equal to the output elasticity of the public spending \((1 - \alpha)\). Contrast to the literature, however, our simple model has a transitional dynamics of the income distribution. Further, since the growth rate also evolves in the process of income convergence, we are able to examine a dynamic relationship between income inequality and economic growth.

The following proposition summarizes the results of this section.

**Proposition 1** In a two class economy of \((h_1, h_2) = (\varepsilon, 1 - \varepsilon)\), there exist two equilibria: an equal economy and a Marxian economy. The equal economy is stable and the Marxian is unstable.

**Proof.** See Appendix. ■
4 Inequality and Growth

4.1 Capital growth

This section analyzes the dynamics of private capital. Our main attention is of course the rate of output growth, but it would be insightful to examine the law of motion of private capital at first because it is likely to be approximate to the law of motion of output.

On a saddle path where \( \dot{\omega}_i = 0 \), the growth rate of capital for household \( i \) is given by

\[
\frac{\dot{k}_i}{k_i} = (1 - \phi)\eta x_i^{-\phi} r - \rho
\]

where \( r \) is given by equation (17). Aggregation gives

\[
\frac{\dot{k}}{k} = \frac{(1 - \phi)\eta r}{\alpha} \left[ \frac{1}{2} (x_1^{-\phi} + x_2^{-\phi}) - (1 - \alpha)x_2^{-\phi} \right] - \rho
\]

(28)

Figure 5 illustrates the loci of \( \dot{k}_1/k_1 \), \( \dot{k}_2/k_2 \) and \( \dot{k}/k \) separately. The solid curve represents \( \dot{k}/k \), and the cross and circle curves represent \( \dot{k}_1/k_1 \) and \( \dot{k}_2/k_2 \), respectively. Starting from around the Marxian economy \( (x_1 = 2\alpha) \), household 2 increases capital investment more than household 1 because the former faces a smaller marginal tax rate. As the income status of household 1 decreases, household 1 also increases capital investment. The growth of aggregate capital is hampered in part by the decreased rental price of capital. In the process of income convergence, the growth of aggregate capital monotonically increases from 2.6 percent to 2.8 percent.

On the other hand, the transition from the fundamentally unequal economy \( (x_1 \approx 0) \) exhibits a non-monotonic pattern. Household 1 increases capital investment sharply not only because he faces a smaller marginal tax rate but also because his income comes from capital only. This pushes up the growth of aggregate capital at first. As the income status of household 1 increases, however, the growth of aggregate capital turns to decrease. The reason is that household 2 decreases capital investment because of the higher marginal tax rate and the reduced rental price of capital. Since the capital share of household 2 is dominant, the negative share effect discourages the growth of aggregate capital. Starting from 2.75 percent level, the growth of aggregate capital rises to about 2.85 percent at about \( x_1 = 0.5 \) and then decreases to 2.8 percent in the long-run equilibrium.

4.2 Output growth

This section examines the transitional growth rate in the process of income convergence. As demonstrated in the previous section, the growth of aggregate capital cannot be monotonic, depending on whether the initial economy is Marxian or fundamentally unequal. It can be seen that the similar result applies to the output growth, although the transitional changes in the rental price of capital make the analysis complicated.
From equations (8) and (19), the rate of output growth is given by

\[ \frac{\dot{y}}{y} = \frac{\ddot{r}}{r} + \frac{\dot{k}}{k} \]  

Differentiating equation (17) with respect to \( x_1 \), and using equation (23), we have the law of motion of the rental price of capital,

\[ \frac{\ddot{r}}{r} = -\frac{1 - \alpha}{\alpha} \frac{\eta(1 - \phi)}{2} \left( x_1 - x_2^\phi \right) x_1 \]

\[ = -\frac{1 - \alpha}{2} [(1 - \phi)\eta]^2 \left[ 1 - \frac{\eta}{2} \left( x_1 - x_2^\phi \right) \right] \frac{1 - 2\alpha}{\alpha} \left( x_1 - x_2^\phi \right)^2 \left( 1 - \frac{x_1}{2\alpha} \right) \]  

Equation (30) implies that \( \ddot{r} = 0 \) when \( x_1 = 0, 1, \) and \( 2\alpha \), and that \( \ddot{r} \) is non-positive for \( \forall x_1 \in [0, 2\alpha] \). Figure 6 illustrates the dynamics of the rental price of capital in the process of income convergence. Although it is a little bit complex, we know the impact of the changes in the price of capital on the output growth is fairly modest.

Substituting equations (28) and (30) into equation (29), we can examine the dynamic relationship between income inequality and economic growth. Figure 7 illustrates the result. Starting from around the Marxian economy \( (x_1 = 2\alpha) \), the growth rate increases monotonically from 2.6 percent to 2.8 percent. Thus, we observe a negative relationship between inequality and growth, which is consistent with empirical evidence in Persson and Tabellini (1994). On the other hand, starting from around the fundamentally unequal economy \( (x_1 = 0) \), the growth rate increases by about one percent until about \( x_1 = 0.6 \), and then decreases. Thus, we observe that the relationship between inequality and growth is at first negative and then positive in the process of income convergence. This inverse U shaped pattern of economic growth supports the Kuznets hypothesis.

5 Discussions

5.1 Welfare

It is well known that the optimal income tax rate is equal to the output elasticity of the public spending in a balanced growth path equilibrium. With respect to the tax progressivity, \( \phi \), we can identify two opposite effects on welfare. Equation (24) shows that tax progressivity reduces the growth rate because it has a negative effect on investment by increasing the marginal tax rate. On the other hand, the reduced investment increases the consumption-capital ratio, which is shown in equations (25) and (26). Since the effects are counteractive,
a natural question is whether there is a positive $\phi$ that maximizes the welfare of at least one of the households. The following proposition answers the question, no.

**Proposition 2** In a two class economy of $(h_1, h_2) = (\varepsilon, 1 - \varepsilon)$, the optimal tax scheme in the equal economy is $(\eta, \phi) = (\alpha, 0)$.

**Proof.** On a balance growth path, the welfare of household 1 is given by

$$u_1 = \frac{1}{\rho} \ln c_1(0) + \frac{\gamma}{\rho^2}$$

Appendix shows that the growth rate and initial consumption are respectively given by

$$\gamma = (1 - \phi)\alpha A^{\frac{1}{\alpha}} \eta (1 - \eta) \frac{1 - \alpha}{\alpha} - \rho$$

$$\frac{c_1(0)}{k_1(0)} = \rho + \alpha A^{\frac{1}{\alpha}} \eta (1 - \eta) \frac{1 - \alpha}{\alpha} \left[ \phi + \frac{2(1 - \alpha)\varepsilon}{1 - 2(1 - \alpha)\varepsilon} \right]$$

Given $\phi < 1$, $\eta = \alpha$ is the unique optimal solution because it maximizes both $\gamma$ and $c(0)$.

Differentiating $u_1$ with respect to $\phi$, we have

$$\frac{\partial u_1}{\partial \phi} = -\frac{\left[\alpha A^{\frac{1}{\alpha}} \eta (1 - \eta) \frac{1 - \alpha}{\alpha}\right]^2}{\rho^2 c_1(0)} \left[ \phi + \frac{2(1 - \alpha)\varepsilon}{1 - 2(1 - \alpha)\varepsilon} \right]$$

which is negative for $\forall \phi \geq 0$ and $\forall \varepsilon \in [0, 1]$. Thus, $\phi = 0$ is the unique optimal solution.

One of the policy implications of our result is an application of the wisdom of the second theorem of welfare economics. Suppose that the societies agree on the degree of income inequality. The government sets a progressive tax scheme and trusts market mechanism to achieve the admissible income distribution. Once the income distribution becomes admissible one, then the government abandons the progressive tax scheme, that is, sets $\phi = 0$. Under a proportional tax scheme, the long-run growth with constant income distribution is sustainable in our model. The optimal $\phi$ would be given by the magnitude of welfare benefit of the speed of convergence relative to the welfare cost of progressive taxation associated with the distortionary effect on capital investment.

### 5.2 Growth regression

One of the main topics in public economics is to answer the following questions. Does some kind of public spending contribute to economic growth? If so, how much is it? While Aschauer (1989a, b) and the following researchers estimate a non-negligible positive effect of public capital on the marginal productivity of private capital, the issue remains controversial.\(^8\) One of the reasons of the disagreement among researchers may be the lack of consideration for income

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\(^8\)See, for example, Pfähler, Hofmann, and Bönte (1996) and Kneller, Bleaney, and Gemmell (1999).
distribution. As shown in the previous section, the transitional growth rate
depends on the income distribution, which suggests the growth effect of public
spending would also vary in the process of income convergence.

Figure 8 illustrates the loci of $\frac{\dot{k}}{k}$ when $\eta = 0.8$ (a solid curve) and $\eta = 0.82$
(a dashed curve). The solid curve lies above the dashed one when $x_1 = 1$
because the growth rate is maximized when $\eta = \alpha$ (We assume $\alpha = 0.8$). Since
the tax rate facing a household who earns the average income is $1 - \eta$, we will
observe that a higher tax rate is related to a higher growth rate, given that
income distribution is fairly equal. This is not true, however, if the income
distribution is much skewed. Specifically, let us suppose that $x_1 < 0.5$. Then
the dashed curve lies above the solid curve, which implies that a lower tax rate
is related to a higher growth rate. This example suggests that the disagreement
on the growth effect of public spending could be attributable to the lack of
consideration for income distribution.

[Figure 8 is here]

6 Concluding comments

In a simple endogenous growth model of heterogeneous households, we analyze
the effects of progressive taxation on a dynamic relationship between economic
growth and income distribution. We assume that the heterogeneity arises from
wealth differences associated with firm ownership and the initial allocation of
capital.

We have two main results. First, a perfectly separated economy between
a capitalist and a worker could be an equilibrium, but it is unstable from a
dynamic point of view. Our result casts doubts on the outcome of a two-class
economy model if it is constructed in a static framework.

Second, the relationship between inequality and growth is not monotonic.
Starting from a Marxian economy, the income distribution becomes more equal
and the growth rate increases over time. Thus, we observe a negative rela-
tionship between inequality and growth. On the other hand, starting from a
fundamentally unequal economy in which poor people do not have equity nor
capital, the growth rate increases at first and then decreases in the process of
income convergence. This implies that the relationship between growth and in-
equality is at first negative and then positive. The latter result could explain the
reason why many developed countries have experienced the inverse U shaped
pattern in the process of economic development.

We admit that our model is highly stylized so that the policy implications
should be treated carefully. However, we believe that a distributional considera-
tion developed in our model is relevant to the advance of macroeconomics since
it could shed light on some controversial issues. One of the implications of our
model is related to the growth regression puzzle. Our result suggests that the
distributional consideration could answer the question of whether and to which
extent the government contributes to economic growth.

It would be misleading to assert that taxation should be progressive in that
it plays a direct role in achieving equal society. As demonstrated in this paper,
the optimal tax scheme is proportional once the economy converges to equal society. Nevertheless, we should recognize that tax progressivity contributes not only to income equality but also to economic growth in a transition process.
Appendix [Two class economy: a general case]

Without loss of generality, let us assume that the distribution of wealth is given by

\[ h_1 = \epsilon \]
\[ h_2 = 1 - \epsilon \]

where \( 0 \leq \epsilon \leq 1 \). In this case, the distribution of income \((x_1, x_2)\) is given by

\[ x_1 = 2(1-\alpha)\epsilon + 2\alpha \frac{k_1}{k} \]
\[ x_2 = 2 - x_1 \]

where \( k = k_1 + k_2 \). Nonnegative conditions require \( 2(1-\alpha)\epsilon \leq x_1 \leq 2[\alpha + (1-\alpha)\epsilon] \).

The dynamics for household \( i \) is characterized by

\[ \dot{c}_i = (1-\phi)G_{1i}^x \dot{r} - \rho 
\]
\[ \dot{k}_i = \eta G_{1i}^x y_i \frac{k_1}{k_i} - \frac{c_i}{k_i} \]

where the rental price of capital, \( r \), is given by equation (17).

Let us define the consumption-capital ratio for household \( i \) by \( \omega_i = c_i/k_i \). Substituting \( y_i = r k_i + \pi h_i \) into equation (A3), we have

\[ \frac{\dot{\omega}_1}{\omega_1} = \omega_1 - \rho - \eta x_1^{-\phi} r \left[ \phi + \frac{2(1-\alpha)\epsilon}{x_1 - 2(1-\alpha)\epsilon} \right] \]
\[ \frac{\dot{\omega}_2}{\omega_2} = \omega_2 - \rho - \eta x_2^{-\phi} r \left[ \phi + \frac{2(1-\alpha)(1-\epsilon)}{x_2 - 2(1-\alpha)(1-\epsilon)} \right] \]

Let us transform a state variable from \( x_1 \) to \( q \equiv x_1 - 2(1-\alpha)\epsilon \). Nonnegative conditions require \( 0 \leq q \leq 2\alpha \). Differentiating equation (A1), and after some manipulations, we have

\[ \frac{\dot{q}}{q} = \frac{\dot{k}_1 - \dot{k}}{k_1 - k} \]
\[ = \frac{k_2}{k} \left( \frac{\dot{k}_1}{k_1} - \frac{\dot{k}_2}{k_2} \right) \]
\[ = \left( 1 - \frac{q}{2\alpha} \right) \left\{ \eta r \left( x_1^{-\phi} - x_2^{-\phi} \right) + \omega_2 - \omega_1 + 2(1-\alpha)\eta r \left[ \frac{\epsilon x_1^{-\phi}}{q} - \frac{(1-\epsilon)x_2^{-\phi}}{2\alpha - q} \right] \right\} \]

where \( x_1 = q + 2(1-\alpha)\epsilon \)
\( x_2 = 2(1-\alpha)(1-\epsilon) + 2\alpha - q \)

The dynamics of \((x_1, x_2, q, \omega_1, \omega_2, r)\) is characterized by equations (A3), (A4), (A5), (A6), (A7), and (17). In the same way as the main body, let
us imagine a phase diagram \((\omega_1, \omega_2)\) on a plane \(q = \dot{q} (0 < \dot{q} < 2\alpha)\). There is a unique unstable equilibrium such as

\[
\begin{align*}
\dot{\omega}_1 &= \rho + \eta \hat{x}_1^{-\phi} \left( \phi + \frac{2(1 - \alpha)\epsilon}{\ddot{q}} \right) \\
\dot{\omega}_2 &= \rho \eta \hat{x}_2^{-\phi} \left( \phi + \frac{2(1 - \alpha)(1 - \epsilon)}{2\alpha - \ddot{q}} \right)
\end{align*}
\]

where an upper bar indicates the corresponding value at \(q = \ddot{q}\).

Substituting them into equation (A5), we know in which direction \(q\) moves at \(q = \ddot{q}\):

\[
\frac{\dot{q}}{q} = (1 - \phi) \eta r \left( x_1^{-\phi} - x_2^{-\phi} \right) \left( 1 - \frac{q}{2\alpha} \right)
\]

Equation (A8) implies that \(\dot{q} = 0\) has two solutions \(q^* \equiv 1 - 2(1 - \alpha)\epsilon\) and \(2\alpha\). The former solution corresponds to \(x_1^* = 1\), which implies that both household 1 and 2 have positive capital and that they have the same amount of income. The latter solution corresponds to the Marxian economy in the main body. Given that \(\alpha > 1/2\), \(q^*\) is globally stable and \(2\alpha\) is unstable. Thus, the transitional dynamics of income distribution follows the saddle path, which is illustrated by a locus of \((\dot{\omega}_1, \dot{\omega}_2, \dot{q})\).

In the long-run equilibrium, the growth rate is given by equation (24), and the resource allocations are given by

\[
\begin{align*}
\frac{c_1}{k_1} &= \rho + \alpha A \frac{(1 - \eta)(1 - \eta^{\frac{1 - \alpha}{\alpha}})}{1 - 2(1 - \alpha)\epsilon} \left( \phi + \frac{2(1 - \alpha)\epsilon}{1 - 2(1 - \alpha)\epsilon} \right) \\
\frac{c_2}{k_2} &= \rho + \alpha A \frac{(1 - \eta)(1 - \eta^{\frac{1 - \alpha}{\alpha}})}{1 - 2(1 - \alpha)(1 - \epsilon)} \left( \phi + \frac{2(1 - \alpha)(1 - \epsilon)}{1 - 2(1 - \alpha)(1 - \epsilon)} \right) \\
\frac{k_2}{k_1} &= \frac{1 - 2(1 - \alpha)(1 - \epsilon)}{1 - 2(1 - \alpha)\epsilon}
\end{align*}
\]
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References


Figure 1: Rental price of capital when the income distribution is \((x, 2-x)\)

\[ r = 0.0, \quad \alpha = 0.8, \quad \eta = 0.8, \quad \phi = 0.1, \quad \rho = 0.04, \quad \text{and} \quad A = 0.25 \]
Figure 2: Phase diagram of \((\omega_1, \omega_2)\) on a plane \(x_1 = x\) \((0 < x < 2\alpha)\)
Figure 3: Saddle path in a space of $(\omega_1, \omega_2, x_1)$

$\alpha = 0.8, \eta = 0.8, \phi = 0.1, \rho = 0.04, \text{ and } A = 0.25$
Figure 4: Income convergence

\[
\alpha = 0.8, \eta = 0.8, \phi = 0.1, \rho = 0.04, \text{ and } A = 0.25
\]
Figure 5: The loci of $\dot{k}/k$ (solid), $\dot{k}_1/k_1$ (cross), and $\dot{k}_2/k_2$ (circle)

$\alpha = 0.8$, $\eta = 0.8$, $\phi = 0.1$, $\rho = 0.04$, and $A = 0.25$
Figure 6: The law of motion of rental price of capital

\[ d\ln(r) = 0 = \alpha, \eta = 0.8, \phi = 0.1, \rho = 0.04, \text{ and } A = 0.25 \]
Figure 7: Inequality and growth

\[ d\ln(y) \]

\[ x_1 \]

\[ \alpha = 0.8, \eta = 0.8, \phi = 0.1, \rho = 0.04, \text{ and } A = 0.25 \]
Figure 8: The loci of $k/k$ when $\eta = 0.8$ (solid) and 0.82 (dashed)

$\alpha = 0.8, \phi = 0.1, \rho = 0.04, \text{ and } A = 0.25$