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Healthy Life Expectancy, Dynamic Efficiency, and a Pareto-improving Subsidy for Long-Term Care

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Abstract

It has been recognized the quality of life is not related to life expectancy at birth but to the Heathy Life Expectancy (HALE) at birth. Does the policy priority concerning long-term care for older people become lower if they become healthier than before? In a theoretical model we show the answer depends on the dynamic efficiency condition. Introducing a subsidy for long-term care is efficient on the Pareto principle if the economy is dynamically inefficient. Our result suggests a proper public policy concerning long-term care is necessary to improve the quality of life not only for older generations but also for younger and future generations.

JEL Classification: E62, H21, J14

Keywords: Healthy life expectancy, Dynamic efficiency, Long-term care, Subsidy

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1 Introduction

Facing a rapid rate of population aging, many developed countries have tackled with a tough task of reforming long-term care for fragile elderly people (OECD (2005)). Among most controversial issues, an intergenerational conflict concerning long-term care would be a central issue to solve. More generous benefits for older people may impose a greater financial burden not only on working generation but also on future generations by discouraging work incentives and economic growth.

[Figure 1 is here]

To deal with the issue in a theoretical model, we have to carefully identify how population aging affects the demand for long-term care. Figure 1 illustrates the relation between life expectancy at birth (LE) and healthy life expectancy at birth (HALE) for 193 countries (WHO (2010)). The horizontal axis measures LE and the vertical axis measures the ratio of HALE to LE. In Japan, one of the most aging countries in the world, the life expectancy is about 83 in 2008 and the healthy life expectancy is about 76 in 2007. On average, Japanese people live for 92 per cent of their lifetime with good health . Figure 1 shows the ratio tends to increase with LE. It suggests using LE as a proxy of population aging would be misleading because old people with good health would not demand for long-term care services so much.

In this paper, we analyze whether and to what extent a subsidy for long-term care improves welfare of each generation in an overlapping generations model with two-sector production (a good production and a formal care production). In the first period of life, individuals work, consume goods, and save for retirement. In the second period, they consume goods and care services. Bequest motives are absent. The care service production is labor intensive relative to the good production. Markets work well. In this setting, we introduce a subsidy for long-term care financed by a wage income tax. We have two analytical results. First, introducing a subsidy for long-term care is Pareto-improving if the economy is dynamically inefficient in a Neoclassical world or if the economy exhibits sustainable growth. Second, the maximum rate of subsidy which makes all the generations better-off decreases with HALE. This implies HALE plays a critical role in the reform of public policy concerning long-term care.

The mechanism is simple. Suppose that the government introduces the tax-subsidy scheme once-and-for-all and keeps it thereafter. The scheme increases the demand for long-term care, which reallocates labor from the good sector toward the care service sector. In the short run, the capital-labor ratio in the good sector increases, which increases the wage rate and decreases the interest rate. In a Neoclassical world, the capital-labor ratio comes back to the initial steady-state. Comparing the long run steady states, both capital and labor employed in the good sector after the reform are smaller than before the reform. This change would improve welfare of future generations if the initial capital stock is overaccumulated, that is, the initial steady state is dynamically inefficient. Next, it can be easily proved that all the generations in the transition process are better-off if the future generations are better-off. Finally, the old people who live in the start period of reform could be better-off if the positive effect of subsidy dominates the negative price effect. The same mechanism works in an endogenous growth setting.

Our paper is related to two field of research. First, we rely on a possibility of dynamic inefficiency. It is well known that a transfer from young to old is Pareto-improving if the population growth rate is larger than the interest rate (Samuelson (1954) and Aaron (1966) among others). This argument had ceased since Abel et al. (1989) suggested most OECD countries are dynamically efficient in its stochastic one sector growth model. In a two-sector growth model, however, van Groezen et al. (2007) shows the economy is more likely to be dynamically inefficient if older people prefer services to good consumption, and that a positive pay-as-you-go tax maximizes long-run welfare in the service economy.[1] The mechanism is similar to ours, but it relies on simulations and does not consider the Pareto optimality.

Second, some researchers examine a possibility of Pareto-improving reform in various circumstances. Much more attention has been paid on public pension reforms (Breyer (1989), Breyer and Straub (1993), Wiedmer (1996), Belan et al. (1998), Wigger (1999, 2001), Gyárfás and Marquardt (2001), and van Groezen et al. (2003)). The other are on unemployment insurance (Corneo and Marquardt (2000)), and on bequest taxation (Grossmann and Poutvaara (2009)). The research on a Pareto-improving reform of long-term care seems fairly scarce.

The structure of the paper is as follows. In section 2 we introduce a basic model and analyze the characteristics of equilibrium. We know the subsidy for long-term care reallocates labor from the good production sector to the care service production sector. One of the merits of the scheme is that it is neutral to the capital-labor ratio in the good production sector, which implies factor prices are not affected in the long run. In Section 3 we examine whether and to what extent introducing the scheme is Pareto-improving. We know all the generations are better-off if the steady-state equilibrium is dynamically inefficient. In Section 4 we extend the basic model to an endogenous growth model. We know the sufficient condition for the Pareto optimality is weakened. The final section concludes the paper.

2 Basic model

2.1 Setup

To make the analysis as simple as possible, we assume a gross rate of population growth is constant $n > 0$. Denoting the population in generation t by N_t, we have $N_{t+1} = nN_t$ for all t.

The utility of an individual in generation t, who is born in period t, is given by

$$u_t = \ln c_{1t} + \beta[\rho \ln c_{2t+1} + (1-\rho)\ln m_{t+1}] \tag{1}$$

where c_{1t}, c_{2t+1}, and m_{t+1} stand for good consumption when young, good consumption when old, and consumption of care service, respectively. $0 < \beta < 1$ is a subjective discount factor, and $0 \leq \rho \leq 1$ stands for healthiness in

[1] Cremers (2006) examines the dynamic efficiency condition in a more general two sector OLG model.

retirement. A higher ρ represents a higher healthy life expectancy at birth as discussed in the previous section.[2]

The budget constraint in the first and the second period are given by

$$
\begin{aligned}
(1-\tau_t)w_t &= c_{1t} + s_t && (2)\\
R_{t+1}s_t &= c_{2t+1} + (1-\theta)p_{t+1}m_{t+1} && (3)
\end{aligned}
$$

where s_t stands for a private saving. w_t, R_{t+1}, p_{t+1} stand for a wage rate in period t, a gross interest rate in period $(t+1)$, and a price of the care service in period $(t+1)$, respectively. $0 \leq \theta < 1$ is a rate of subsidy for the purchase of care service , and $0 \leq \tau_t < 1$ is a tax rate.

From equations (2) and (3), the lifetime budget constraint is given by

$$
(1-\tau_t)w_t = c_{1t} + \frac{c_{2t+1} + (1-\theta)p_{t+1}m_{t+1}}{R_{t+1}} \qquad (4)
$$

The optimization problem for an individual in generation t is to maximize equation (1) subject to equation (4). The first-order conditions require

$$
\begin{aligned}
\frac{1}{c_{1t}} &= \lambda_t\\
\frac{\beta\rho}{c_{2t+1}} &= \frac{\lambda_t}{R_{t+1}}\\
\frac{\beta(1-\rho)}{m_{t+1}} &= \frac{\lambda_t(1-\theta)p_{t+1}}{R_{t+1}}
\end{aligned}
$$

where λ_t stands for a multiplier attached to equation (4).

Solving them, we have net demand functions such as

$$
\begin{aligned}
c_{1t} &= \frac{(1-\tau_t)w_t}{1+\beta} && (5)\\
c_{2t+1} &= \frac{\beta\rho}{1+\beta}(1-\tau_t)w_t R_{t+1} && (6)\\
m_{t+1} &= \frac{\beta(1-\rho)}{1+\beta}\frac{(1-\tau_t)w_t R_{t+1}}{(1-\theta)p_{t+1}} && (7)\\
s_t &= \frac{\beta}{1+\beta}(1-\tau_t)w_t && (8)
\end{aligned}
$$

Note that

$$
\begin{aligned}
c_{2t+1} &= \rho R_{t+1}s_t && (9.1)\\
m_{t+1} &= \frac{(1-\rho)R_{t+1}s_t}{(1-\theta)p_{t+1}} && (9.2)
\end{aligned}
$$

[2] More generally, the utility in the second period may be

$$
v_{t+1} = \rho(\ln c^g_{2t+1} + \ln m^g_{t+1}) + (1-\rho)(\ln c^b_{2t+1} + \ln m^b_{t+1})
$$

where a superscript g stands for good health and b stands for bad health. Let us define composite consumption of good and service by

$$
\begin{aligned}
c_{2t+1} &= c^g_{2t+1}\left(c^b_{2t+1}\right)^{\frac{1-\rho}{\rho}}\\
m_{t+1} &= \left(m^g_{t+1}\right)^{\frac{\rho}{1-\rho}} m^b_{t+1}
\end{aligned}
$$

Then, the utility above is reduced to equation (1).

The retirement income $R_{t+1}s_t$ is optimally allocated between the expenditures of good consumption c_{2t+1} and care service $(1-\theta)p_{t+1}m_{t+1}$. Equations (9.1) and (9.2) imply the healthiness in retirement, ρ, plays a critical role in the allocation of labor between the good production sector and the care service sector.

The technology in the good production sector is specified by a Cobb-Douglas form such as

$$Y_t^g = F(K_t, L_t^g) = AK_t^\alpha (L_t^g)^{1-\alpha}$$

Assuming factor markets are competitive, the factor prices are given by

$$w_t = (1-\alpha)AK_t^\alpha (L_t^g)^{-\alpha} \tag{10}$$
$$R_t = \alpha AK_t^{\alpha-1}(L_t^g)^{1-\alpha} \tag{11}$$

It seems reasonable to assume the technology in the care service sector is labor-intensive relative to the good production sector. To simplify the analysis, we assume one unit of labor produces one unit of care service in the care service sector. The production function is

$$Y_t^m = f(L_t^m) = L_t^m$$

Competition in the labor market makes the price of care service and the wage rate equal,

$$p_t = w_t \tag{12}$$

The balanced budget of tax-subsidy scheme requires

$$N_t \tau_t w_t = N_{t-1}\theta p_t m_t \tag{13}$$

The left-hand side is the revenue collected from young generation in period t, and the right-hand side is the expenditure paid to old generation in period t.

There are four markets in this economy, i.e., the market of labor, capital, care service, and good. The market clearing conditions are respectively given by

$$N_t = L_t^g + L_t^m \tag{14}$$
$$K_{t+1} = N_t s_t \tag{15}$$
$$Y_t^m = N_{t-1}m_t \tag{16}$$
$$Y_t^g = N_t c_{1t} + N_{t-1}c_{2t} + K_{t+1} \tag{17}$$

where one of them is redundant to solve the equilibrium by Walras law.

2.2 Equilibrium

In this subsection, we derive (i) the allocation of labor between the two sectors, (ii) the required contribution rate to balance the budget, (iii) the law of motion of capital, and (iv) a condition for the dynamic efficiency at a steady state.

First, let us define the labor share in the good sector by

$$\gamma_t = \frac{L_t^g}{N_t} \tag{18}$$

The labor share in the care service sector is $L_t^m/N_t = 1-\gamma_t$. First, we have the following lemma.

Lemma 1 *The labor share in the good sector is constant over time and given by*

$$\gamma_t = \frac{(1-\alpha)(1-\theta)}{\alpha(1-\rho)+(1-\alpha)(1-\theta)} \tag{19}$$

which is increasing in ρ, and decreasing in θ and α.

Proof. Substituting equation (9.2) into equation (16) with $Y_t^m = L_t^m$,

$$\begin{aligned} L_t^m &= \frac{(1-\rho)R_t N_{t-1} s_{t-1}}{(1-\theta)p_t} \\ &= \frac{(1-\rho)R_t K_t}{(1-\theta)p_t} \end{aligned}$$

where the second equality comes from equation (15).

Note that the ratio of capital income to labor income in the good sector is constant over time,

$$\frac{R_t K_t}{w_t L_t^g} = \frac{\alpha}{1-\alpha}$$

Substituting this and equation (18) into the equation above, we have

$$1-\gamma_t = \frac{1-\rho}{1-\theta}\frac{\alpha}{1-\alpha}\gamma_t$$

which gives equation (19). ■

Note that per capita income in period t is given by

$$\begin{aligned} y_t &= \frac{Y_t^g + p_t Y_t^m}{N_t} \\ &= A\left(\frac{K_t}{N_t}\right)^{\alpha}\left[\alpha(\gamma_t)^{1-\alpha}+(1-\alpha)(\gamma_t)^{-\alpha}\right] \end{aligned}$$

Given K_t and N_t, the per capita income decreases with γ_t because a smaller labor share in the good sector pushes up wages, which dominates a loss of capital income.

Second, we derive the relation between the subsidy rate and the tax rate. Substituting equations (12) and (16) with $Y_t^m = L_t^m$ into equation (13), we have

$$\tau_t = \theta(1-\gamma_t) = \frac{\alpha(1-\rho)\theta}{\alpha(1-\rho)+(1-\alpha)(1-\theta)} \tag{20}$$

which is obviously increasing in the subsidy rate θ.

Third, let us define the capital-labor ratio in the good sector by

$$k_t = \frac{K_t}{L_t^g}$$

The following lemma states the law of motion of capital.

Lemma 2 *The capital-labor ratio in the good sector follows*

$$k_{t+1} = \Gamma k_t^{\alpha}$$

where

$$\Gamma = \frac{\beta(1-\rho\alpha)A}{n(1+\beta)} \tag{21}$$

Given $k_0 = K_0/(\gamma N_0)$, k_t converges monotonically to a unique steady state $k_\infty = \Gamma^{\frac{1}{1-\alpha}}$. The public scheme affects the initial value, k_0, but does not affect the steady state value.

Proof. Substituting equation (8) into equation (15), we have

$$k_{t+1} = \frac{\beta(1-\tau_t)w_t}{n(1+\beta)\gamma_{t+1}}$$

Substituting equations (10), (19), and (20) into this, we have equation (21).

■

The neutrality of the subsidy policy relies on our specification of preference and production technologies. On the one hand, the subsidy for care service decreases employment in the good sector, which increases the capital-labor ratio. On the other hand, the related wage income tax reduces private savings, which decreases the capital-labor ratio. The opposite effects are exactly cancelled out in our specification.

Finally, we derive a condition for the dynamic efficiency at a steady state. From equations (11) and (21), the gross interest rate is given by

$$R_\infty = \alpha A k_\infty^{\alpha-1} = \frac{\alpha(1+\beta)}{\beta(1-\rho\alpha)}n$$

The steady state equilibrium is dynamically (in)efficient if $R_\infty > (<) n$. If $R_\infty = n$, the steady state capital stock satisfies the Golden Rule. Specifically, we have the following lemma.

Lemma 3 *The steady state equilibrium is dynamically (in)efficient if*

$$\rho > (<) \frac{1-\alpha}{\alpha} - \frac{1}{\beta} \equiv \hat{\rho} \tag{22}$$

$\hat{\rho}$ is increasing in β and decreasing in α. The investment ratio is higher when individuals are more patient, and/or when the income share of labor $(1-\alpha)$ is larger.[3] Therefore a larger ρ is necessary to satisfy the dynamic efficiency condition for a larger β and/or for a smaller α.[4]

[3] It can be shown the investment ratio is constant over time and given by

$$\frac{K_{t+1}}{Y_t^g} = \frac{\beta}{1+\beta}(1-\rho\alpha)$$

[4] Assuming that an annual discount rate is 1.5 percent, and that one period is 35 years, we have $\beta = 1.015^{-35} = 0.59$. Under a plausible assumption that the income share of labor is 70 per cent ($\alpha = 0.3$), we have $\hat{\rho} = 0.65$. Assuming that individuals enter the economy at age 20, the life expectancy is $20 + 35 * 2 = 90$, and the healthy life expectancy when $\rho = \hat{\rho}$ is $20 + 35 * 1.65 = 77.7$.

3 Pareto-improving reform

In this section we examine whether introducing a subsidy for long-term care improves welfare of each generation. Suppose that the economy without the subsidy stays at the steady state in period 0. Then the government introduces the scheme once-and-for-all in period 0, and keeps it thereafter. We examine the welfare of (i) future generations, (ii) generations in the transition process, and (iii) the old generation in period 0. If they become better-off, introducing the scheme is welfare-improving.

[Figure 2 is here]

Figure 2 illustrates the effect of introducing the public scheme on the accumulation of capital. Before the reform, the economy stays at point A. After the reform, the capital-labor ratio in the good sector jumps to k_0 because the employment in the good sector decreases. Since the capital-labor ratio is higher than the steady state value, capital decreases over time according to equation (21). In the long run, the capital-labor ratio comes back to the initial steady state, but capital itself is smaller than that before the reform. In this model, public subsidies for long-term care suppress the accumulation of capital both in the short run and in the long run.

3.1 Future generations

In the long run, the individual welfare is given by

$$u = \ln c_1 + \beta[\rho \ln c_2 + (1-\rho)\ln m] \tag{23}$$

The wage rate and the interest rate after the reform are the same as those before the reform in the long run. Thus, we have

$$\begin{aligned} c_1 &= (1-\tau)\hat{c}_1 \\ c_2 &= (1-\tau)\hat{c}_2 \\ m &= \frac{1-\tau}{1-\theta}\hat{m} \end{aligned}$$

where $\hat{c}_1$, $\hat{c}_2$, and $\hat{m}$ stand for the steady state values before the reform.

The long run welfare effect of the public long-term care is (i) positive because the demand for formal care increases, and (ii) negative because both consumption when young and old decrease.

Let us define the welfare difference as

$$\varphi(\theta) = u - \hat{u}$$

where $\hat{u}$ stands for a steady state welfare before the reform.

From equation (23), we have

$$\begin{aligned} \varphi(\theta) &= (1+\beta)\ln(1-\tau) - \beta(1-\rho)\ln(1-\theta) \\ &= (1+\beta\rho)\ln(1-\theta) + (1+\beta)\ln\frac{1-\rho\alpha}{\alpha(1-\rho)+(1-\alpha)(1-\theta)} \end{aligned} \tag{24}$$

The second equality comes from equation (20). We know $\varphi(0) = 0$. If $\varphi'(0) > 0$, then introducing the scheme is welfare-improving in the long run. Otherwise, it worsens welfare. Specifically, we have the following proposition.

Proposition 4 *Introducing the tax-subsidy scheme improves welfare of future generations if the steady state equilibrium is dynamically inefficient. The optimal rate of subsidy for future generations is given by*

$$\theta^* = 1 - \frac{\alpha(1+\beta\rho)}{\beta(1-\alpha)} \tag{25}$$

which is increasing in β and decreasing in ρ and α.

If the economy is dynamically efficient, then the reform worsens the welfare of future generations.

Proof. Differentiating equation (24) with respect to θ, we have

$$\varphi'(\theta) = \frac{(1-\rho)[\beta(1-\alpha)(1-\theta) - \alpha(1+\beta\rho)]}{(1-\theta)[\alpha(1-\rho) + (1-\alpha)(1-\theta)]}$$

$\varphi'(0) > 0$ is equivalent to equation (22). Solving $\varphi'(\theta) = 0$, we have equation (25). ■

3.2 Generations in transition

First, note that the capital-labor ratio in the good sector decreases monotonically to a unique steady state and that the steady state is the same as that before the reform,

$$k_0 > k_1 > \cdots > k_\infty = \hat{k} \tag{26}$$

where $\hat{k}$ stands for the initial capital-labor ratio.

The welfare of generation $t \geq 0$ is given by

$$u_t = \ln c_{1t} + \beta[\rho \ln c_{2t+1} + (1-\rho)\ln m_{t+1}]$$

where c_{1t}, c_{2t+1}, and m_{t+1} are given by the following lemma.

Lemma 5 *The consumption of good and care service are given as*

$$c_{1t} = \frac{1-\alpha}{1+\beta}(1-\tau)Ak_t^\alpha \tag{27.1}$$

$$c_{2t+1} = \rho\alpha n\gamma A\Gamma^\alpha k_t^{\alpha^2} \tag{27.2}$$

$$m_{t+1} = \frac{(1-\rho)\alpha n\gamma}{(1-\alpha)(1-\theta)} \tag{27.3}$$

where Γ and γ are respectively given by equations (19) and (21).

Proof. Substituting equations (10) and (18) into equation (5), we have equation (27.1).

Substituting equation (15) into equation (9.1), and using $R_tK_t/(w_tL_t^g) = \alpha/(1-\alpha)$, we have

$$c_{2t+1} = \frac{\rho\alpha n\gamma}{1-\alpha}w_{t+1}$$

From equations (10) and (21), we know $w_{t+1} = (1-\alpha)Ak_{t+1}^{\alpha} = (1-\alpha)A(\Gamma k_t^{\alpha})^{\alpha}$. Substituting this into the equation above, we have equation (27.2).

Substituting equations (12) and (15) into equation (9.2), and using $R_t K_t/(w_t L_t^g) = \alpha/(1-\alpha)$, we have equation (27.3). ■

Equations (27.1)-(27.3) give the welfare of generation t such as

$$u_t = \ln(1-\tau) + \beta\ln\gamma - \beta(1-\rho)\ln(1-\theta) + \alpha(1+\beta\rho\alpha)\ln k_t$$

where we omitted the constant terms.

Substituting γ in equation (19) and τ in equation (20) into this, we have (by omitting the constant terms)

$$u_t = (1+\beta\rho)\ln(1-\theta) - (1+\beta)\ln[\alpha(1-\rho)+(1-\alpha)(1-\theta)] + \alpha(1+\beta\rho\alpha)\ln k_t \quad (28)$$

Comparing equations (24) and (28), we are able to prove the optimal subsidy rate θ^* in equation (25) is also optimal for generation $t \geq 0$ except for the capital effect. We know that k_t is larger than the value before the reform (see equation (26)), and that the impact of capital stock on welfare is positive (see equation (28)). Therefore, we have the following proposition.

Proposition 6 *If the future generations are better-off by introducing the tax-subsidy scheme, then all the generations in the transition process are also better-off.*

3.3 The old generation at the reform

The welfare of generation -1 in period 0 is given by

$$v_0 = \rho\ln c_{20} + (1-\rho)\ln m_0 \quad (29)$$

where

$$R_0 s_{-1} = c_{20} + (1-\theta)p_0 m_0$$

Note that s_{-1} is predetermined. The welfare effect is (i) positive because the purchase of formal care is subsidized, and (ii) negative because the retirement income decreases and the price of formal care (the wage rate) increases by the general equilibrium effect.

After the reform, old individuals in generation -1 reallocate retirement income between good consumption and care service consumption. Solving the problem, we have

$$\begin{aligned} c_{20} &= \rho R_0 s_{-1} \\ m_0 &= \frac{(1-\rho)R_0 s_{-1}}{(1-\theta)p_0} \end{aligned}$$

Note that

$$\begin{aligned} p_0 &= w_0 = (1-\alpha)AK_0^{\alpha}(\gamma N_0)^{-\alpha} \\ R_0 &= \alpha AK_0^{\alpha-1}(\gamma N_0)^{1-\alpha} \end{aligned}$$

Substituting them into equation (29), we have (by omitting the constant terms)

$$v_0 = (1-\rho\alpha)\ln\gamma - (1-\rho)\ln(1-\theta) + (1-\rho\alpha)(\ln N_0 - \ln K_0) + \ln s_{-1} \quad (30)$$

A second term in equation (30) stands for a direct positive effect related to the subsidy for formal care, and a first term stands for the indirect negative effect related to changes in factor prices (note that $\partial\gamma/\partial\theta < 0$). If the positive effect is dominant, then introducing the public scheme improves welfare. Otherwise, it worsens welfare. Specifically, we have the following proposition.

Proposition 7 *Introducing the tax-subsidy scheme strictly improves welfare of the old generation at the reform. The optimal subsidy rate is given as*

$$\theta^{**} = 1 - \rho\alpha \tag{31}$$

Proof. Substituting γ in equation (19) into equation (30), and differentiating it with respect to θ, we have

$$\frac{\partial v_0}{\partial \theta} = \frac{(1-\alpha)(1-\rho)(1-\rho\alpha-\theta)}{(1-\theta)[\alpha(1-\rho)+(1-\alpha)(1-\theta)]}$$

which gives equation (31). ■

3.4 Result

Comparing equations (25) and (31), we have:

Lemma 8 *The optimal subsidy rate for future generations is less than that for the old generation at the reform, $\theta^* < \theta^{**}$.*

With this lemma, and Proposition 4, 6, 7, we conclude:

Proposition 9 *Introducing the tax-subsidy scheme is Pareto-improving if the steady state is dynamically inefficient. Increasing the subsidy rate until θ^* in equation (25) is also Pareto-improving.*

[Figure 3 is here]

Figure 3 illustrates the optimal rate of subsidy for future generations, θ^*, and that for the old generation at the reform, θ^{**}. The horizontal axis measures the healthiness in retirement, ρ. The steady state is dynamically inefficient if $\rho < \hat{\rho}$, and dynamically efficient if $\rho > \hat{\rho}$. For a $\rho < \hat{\rho}$, there exists a subsidy rate $\theta \leq \theta^*$ such that it achieves Pareto improvement. The more health status in old-age improves, the smaller the optimal subsidy is.

4 Discussions

In this section we extend the basic Neoclassical growth model to an endogenous growth model with capital externality (Arrow (1962) and Romer (1988)). We show that the result obtained in the previous section is robust if the economy exhibits sustainable growth.

The production function in the good sector is modified as

$$Y_t^g = F(K_t, B_t L_t^g) = AK_t^{\alpha}(B_t L_t^g)^{1-\alpha}$$

where B_t is a labor-augmenting technology and specified by

$$B_t = \frac{K_t}{L_t^g} \tag{32}$$

With equation (32), the good output and factor prices are respectively given by

$$\begin{aligned} Y_t^g &= AK_t \\ w_t &= (1-\alpha)\frac{AK_t}{L_t^g} \\ R_t &= \alpha A \end{aligned}$$

The per capita income in period t, $y_t = (Y_t^g + p_t Y_t^m)/N_t$, is given by

$$y_t = A\kappa_t \left(\alpha + \frac{1-\alpha}{\gamma_t}\right)$$

where γ_t is the labor share in the good sector, and κ_t is the over-all capital-labor ratio,

$$\kappa_t = \frac{K_t}{N_t}$$

In the same way as the basic model, the per capita income decreases with γ_t for a given κ_t. It can be easily proved that the labor share γ_t is given by equation (19), and that the tax rate τ_t is given by equation (20).

The growth rate of per capita income is

$$\frac{y_{t+1}}{y_t} = \frac{\kappa_{t+1}}{\kappa_t} = \frac{\beta(1-\rho\alpha)A}{n(1+\beta)} = \Gamma \tag{33}$$

which implies the tax-subsidy scheme is neutral to the growth rate.

Let us consider a public reform in the same way as the basic model.

The welfare of generation $t \geq 0$ is given by

$$u_t = \ln c_{1t} + \beta[\rho \ln c_{2t+1} + (1-\rho)\ln m_{t+1}]$$

where

$$\begin{aligned} c_{1t} &= \frac{(1-\alpha)A}{1+\beta}\frac{1-\tau_t}{\gamma_t}\kappa_t \\ c_{2t+1} &= \rho\alpha n A \Gamma \kappa_t \\ m_{t+1} &= \frac{(1-\rho)\alpha n \gamma_{t+1}}{(1-\alpha)(1-\theta)} \end{aligned}$$

By omitting the constant terms, we have

$$\begin{aligned} u_t &= \ln(1-\tau) - [1-\beta(1-\rho)]\ln\gamma - \beta(1-\rho)\ln(1-\theta) + (1+\beta\rho)\ln\kappa_t \\ &= -\beta(1-\rho)\ln[\alpha(1-\rho) + (1-\alpha)(1-\theta)] + (1+\beta\rho)\ln\kappa_t \end{aligned}$$

which implies

$$\frac{\partial u_t}{\partial \theta} > 0$$

for $\forall t \geq 0$. The reason is that m_{t+1} increases with θ while c_{1t} and c_{2t+1} are not affected by θ for a given κ_t. Since the per capita capital growth is neutral to θ, the increased formal care caused by the scheme improves the welfare of all the generations after the reform.

The welfare of generation -1 in period 0 is

$$v_0 = \rho \ln c_{20} + (1-\rho) \ln m_0$$

where

$$\begin{aligned} c_{20} &= \rho R_0 s_{-1} \\ m_0 &= \frac{(1-\rho) R_0 s_{-1}}{(1-\theta) p_0} \end{aligned}$$

Note that

$$\begin{aligned} p_0 &= \frac{(1-\alpha) A \kappa_0}{\gamma} \\ R_0 &= \alpha A \\ \gamma &= \frac{(1-\alpha)(1-\theta)}{\alpha(1-\rho) + (1-\alpha)(1-\theta)} \end{aligned}$$

Substituting them into v_0, we have

$$v_0 = -(1-\rho) \ln[\alpha(1-\rho) + (1-\alpha)(1-\theta)] - (1-\rho) \ln \kappa_0 + \ln s_{-1}$$

which implies

$$\frac{\partial v_0}{\partial \theta} > 0$$

Two effects are recognized. First, the subsidy increases care service consumption. Second, the price of care service increases. The former effect dominates the latter effect, so that introducing the tax-subsidy scheme improves the welfare of generation -1.

We conclude introducing the tax-subsidy scheme is Pareto-improving because all the generations are better-off. It is not the dynamic efficiency condition but a condition for positive growth, $\Gamma > 1$, that is required for sustainable growth.[5]

5 Concluding remarks

In this paper we show introducing a subsidy for long-term care is Pareto-improving if the economy is dynamically inefficient in a Neoclassical world or if the economy exhibits sustainable growth. The maximum rate of subsidy which

[5] Since the per capita income grows at a rate Γ, the dynamic efficiency condition is modified as

$$R \gtreqless \Gamma n \Leftrightarrow \rho \gtreqless \hat{\rho}$$

makes all the generations better-off decreases with the healthy life expectancy at birth (HALE). Our results implies HALE plays a critical role in the reform of public policy concerning long-term care. Since our model is highly stylized and based on strong assumptions, the policy implication should be treated carefully. However, our message would be useful for a debate on reforming long-term care. A proper public policy concerning long-term care is necessary to improve the quality of life not only for older generations but also for younger and future generations.

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Figure 1 Life expectancy (LE) and Healthy life expectancy (HALE)

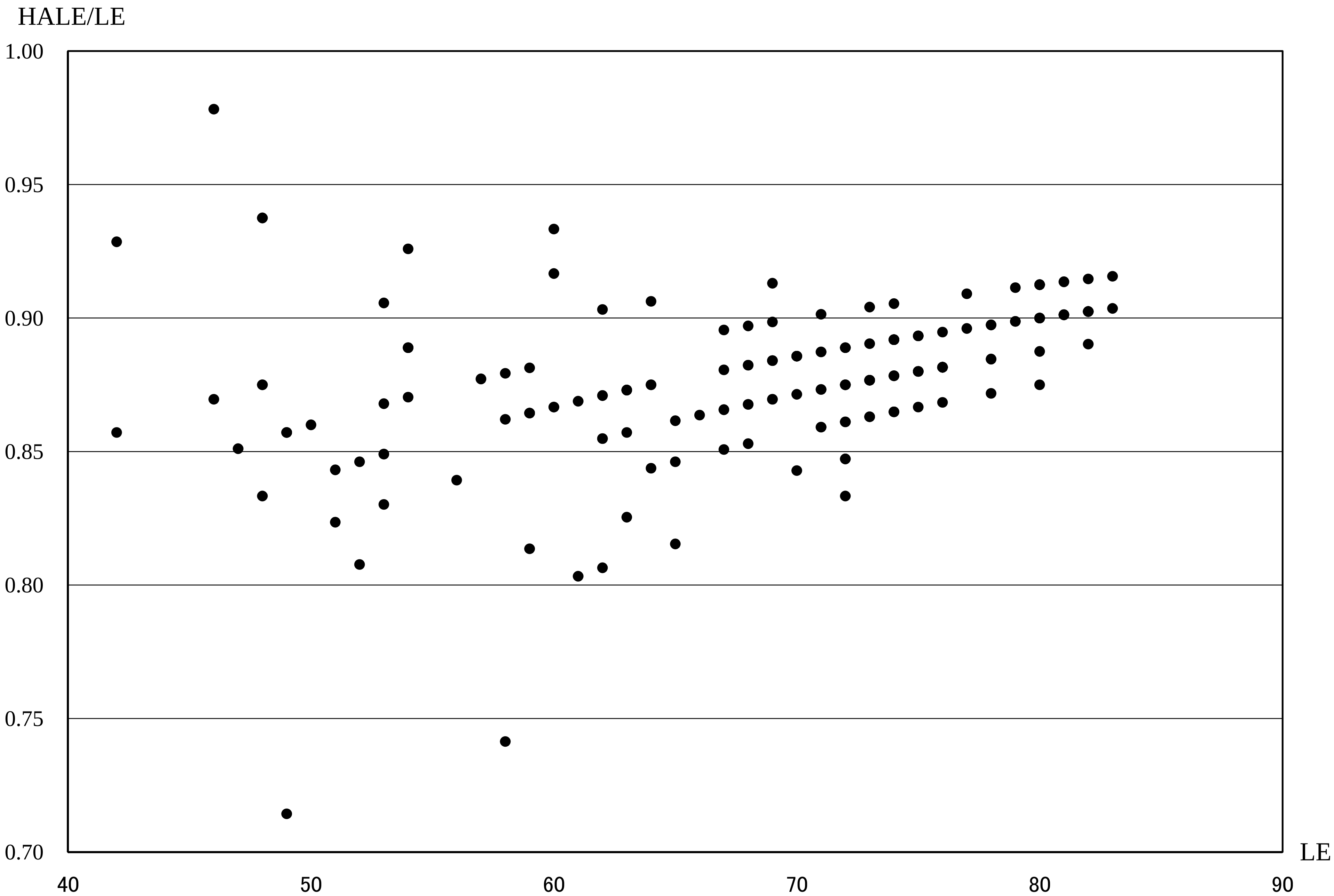

Figure 2 Dynamics

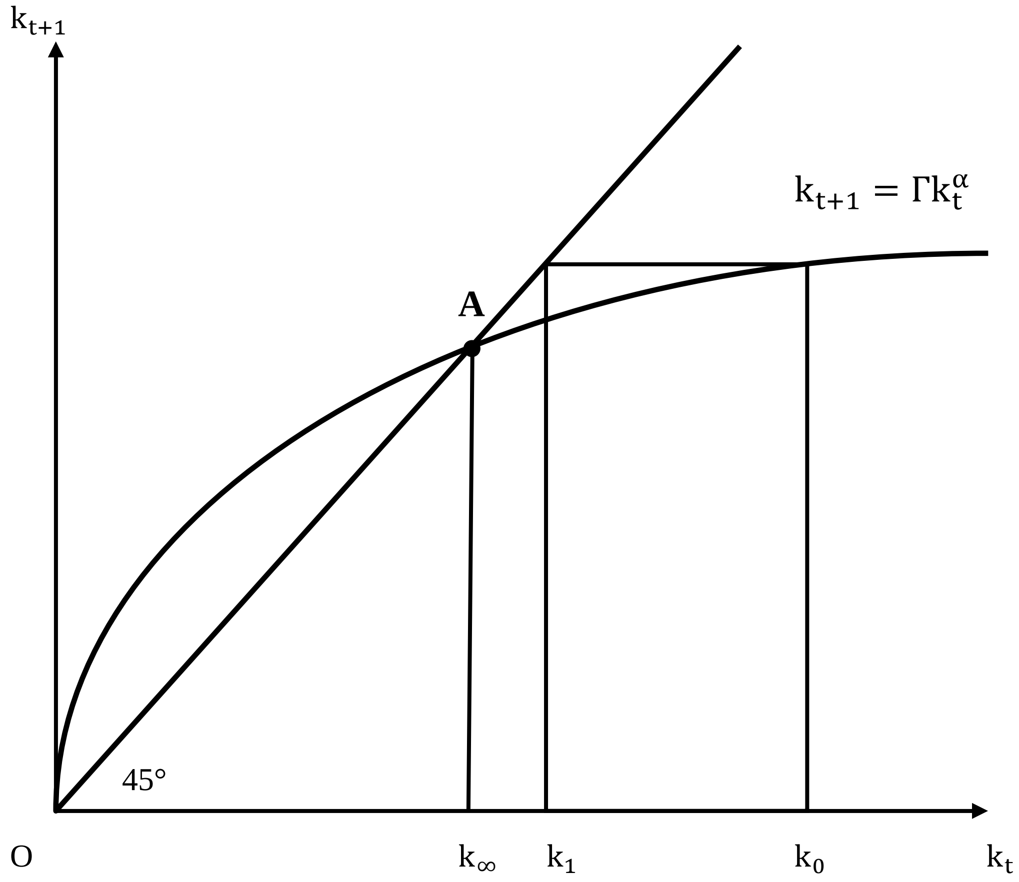

Figure 3　Optimal subsidy