Derivative Meaning in Graphical Representations

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Abstract

This paper reports on the phenomenon that may be called "derivative meaning," where the basic semantic conventions for certain graphical representation systems give rise to additional informational relations between features of representations and features of the represented. We will discuss several examples of graphical systems, such as the systems of scatter plots, data maps, and tabular representations, whose informational potentials heavily depend on this phenomenon. We will then give an analysis of the way a new meaning relation is derived from basic semantic conventions, and specify the exact conditions for a representation system to support this phenomenon.

1. Introduction

The two charts in Figure 1 are the results of presenting the two sets of data in Table 1 in the form of scatter plots. The example is borrowed from Tufte [12] with slight modifications.

I		II	
X	Y	X	Y
10.0	8.04	10.0	7.46
8.0	6.95	8.0	6.77
13.0	7.58	13.0	12.74
9.0	8.81	9.0	7.11
11.0	8.33	11.0	7.81
14.0	9.96	14.0	8.84
6.0	7.24	6.0	6.08
4.0	4.26	4.0	5.39
12.0	10.84	12.0	8.15
7.0	4.82	7.0	6.42
5.0	5.68	5.0	5.73

Table 1. Two Sets of Data

On the one hand, each of these scatter plots is "intertranslatable" with the table of the corresponding number: one provides sufficient information to reproduce the other, and vice versa. In this limited sense, each plot has the same informational content as the corresponding table.

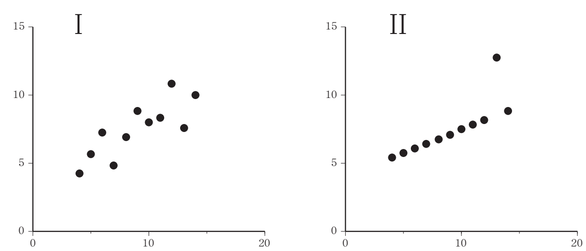

Figure 1. Scatter Plots for Table 1

On the other hand, there is a definite sense in which each plot reveals more information than the table does. The particular shape formed by the dots on a scatter plot seems to indicate some general fact about the data, or more precisely, about the situation that the data are about. Thus, the particular shape appearing in plot I indicates that Y-values and X-values are positively correlated, and the shape in plot II indicates that Y-values and X-values are mostly proportional.

In fact, it is due to this additional informational relation that we sometimes prefer scatter plots to simple tabular displays. Kosslyn [5] makes clear this function of scatter plots:

> Scatter plots...employ point symbols (such as dots, small triangles, or squares) as content elements. The height of each point symbol indicates an amount. These displays typically include so many points that they form a cloud; information is conveyed by the shape and the density of the cloud. (p. 46.)

It is clear, and almost trivial, that the particular shape and the density of the cloud formed in some scatter plot carries information that the corresponding table of data would not. In fact, this type of additional informational relations are quite prevalent in graphical representation systems, and as the above example illustrates, their existence is often the very reason why a given system is more effective than others as a method of displaying certain set of information. Nevertheless, when it comes to the question how such an additional informational relation arises in a given representation system, things are much less clear. The aim of this paper is to give an detailed answer to this general question.

In the next section, we will argue that the informational relation in question can *not* be simply explained as a part of the basic semantic conventions associated with this system of scatter plots. It is more appropriate to consider the relation as something that *naturally* holds, *given* the basic semantic conventions. Drawing on this argument, we will introduce the general notion of "derivative meaning" supported by a graphical system. In section 3, we will show that the phenomenon is in fact general, discussing several instances of graphical systems that support derivative meaning. They include the systems of line graphs, star graphs, data maps, node-edge graphs, and even tabular representations. In section 4, we will turn to the task of formally analyzing the phenomenon. After demonstrating how derivative meaning is in fact derivable from the basic semantic conventions, we will spell out the conditions for a representation system to support derivative meaning. Such a specification is important, partly because it shows that derivative meaning is *not* an arbitrary matter, whose justification totally depends on the person who interprets a representation. Finally, section 5 briefly discusses the relationship of this work to some other works in formal semantics of graphical representations, especially, those in the model-theoretic framework and in the channel-theoretic framework.

2. Phenomenon of Derivative Meaning

Generally, the meaning of a graphical representation is determined on the basis of the set of semantic conventions associated with it. We can express these conventions in the form of implication, which is supposed to hold if a representation is accurate. For example, the semantic conventions for a system of Venn diagrams could be expressed in the following way:

- If a variable appears in the intersection of two circles, then the sets denoted by them have a non-empty intersection,
- If the intersection of two circles is shaded, then the sets denoted by them are disjoint.

These semantic relations are "conventional" in the sense that the initial decisions that established them are essentially arbitrary. For example, shading in a Venn diagram means emptiness in a different way than smoke in sky means fire below. The latter informational relation is based on more or less reliable natural law, while the former is rooted in John Venn's initial decision on what feature of his diagrams to mean what features of the depicted object.

Now the informational relation that we saw between the particular shapes and densities of dots on scatter plots and the general facts indicated by them do not seem to be purely conventional in this sense. In our view, the most fundamental semantic convention associated with scatter plots is the following:

(1) If a dot appears at the Y-coordinate m and the X-coordinate n, then there is an instance in the data with the X- value and the Y-value represented by n and m respectively.

In fact, this rule scheme is all that we must consult in order to draw scatter plots from the data in Table 1. Furthermore, this rule is purely a matter of convention, rooted in the initial decision that the inventor of scatter plots made a long time ago. In contrast, the informational relation that we identified earlier seems to be in different status. Take, for example, the following informational relation underlying the interpretation of plot I that we gave earlier.

(2) If dots form a left-slanted pattern, the X-values and the Y-values of the instances in the data are positively correlated.

It is not necessary for the establishment of this relationship that somebody has explicitly declared it as a semantic convention associated with scatter plots. The relationship (2) is something that naturally holds once the basic semantic convention such as (1) has been adopted. It is partly *derivative*, and is not a part of the *primitive* semantic conventions associated with scatter plots.

The intuition that the informational relations such as (2) are not a part of the primitive semantic conventions is further confirmed by the fact that they are *discoverable*. It often happens that one who knows the basic semantic rules of scatter plots discovers a new way of reading them, realizing that a particular informational relation between a pattern of dots and a general fact about the presented data. Thus we often talk about "experts" of reading scatter plots, to refer to those who are familiar with these additional informational relations supported by scatter plots. Now, it would be quite counter-intuitive to say that each of these newly discovered informational relations is a part of the primitive semantic conventions for scatter plots. For if that were the case, scatter plots would come with a quite complex set of semantic conventions, and nobody but a few experts could claim that he or she is familiar with the semantics of scatter plots.

3. Examples

The main concern of this paper is this type of *derivative* informational relation supported by a representation system, namely, the informational relation that is somehow derivable from, yet not a part of, the primitive semantic conventions of the system. Once we pay attention to this phenomenon, we easily find it quite common in various graphical modes of representation. Let us explore some of those examples, to get a sure grip of the phenomenon that we are trying to explain.

3.1. Line Graph

Just as dots in a scatter plot form a particular cloud that carries information, data points in a line graph forms a informative slope or curve when connected by lines. For example, Kosslyn [5] explains the utility of the line graph in Figure 2 in the following way:

> Here it is apparent that all groups but one show lower levels of "parafabuloid" with increased age—women in the lower income group have the reverse trend. Spotting this trend the table is difficult, but seeing in this graph is easy: Differences in the orientations of the lines convey the different trends, and the eye and mind quickly register such differences. (p. 10.)

Kosslyn is appealing to the following informational relation supported by this particular system of line graphs:

(3) If there is a single leftward slope among rightward slopes, all groups but one has lower levels of parafabuloid with increased age.

It is due to this informational relation that the line graph lets us "spot" the relevant trend of parafabuloid levels.

Yet (3) is an informational relation that we would call "derivative." The primitive semantic conventions for line graphs are what can be summarized in the following way.

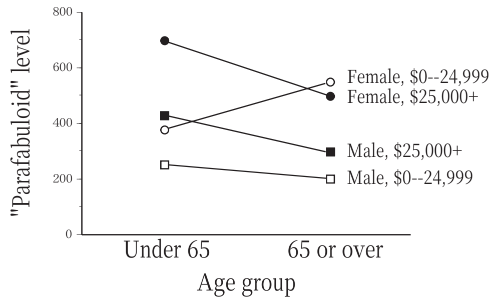

Figure 2. A Line Graph

(4) If a line passes the Y-coordinate m at the X-coordinate n, then the parameter represented by the line has the Y-value represented by m at the X-value represented by n.

The informational relation (3) is not a part of the primitive semantic conventions summarized in (4). Rather, it is a relation that naturally holds once the latter conventional relation is established.

Yet this type of derivative informational relations significantly contribute to the overall informational utility of line graphs. In Kosslyn's words, "Patterns of lines can signal specific information for readers who have had experience with similar line graphs (and so have appropriate knowledge)" (p. 34, [5]).

3.2. Star Graph

Perhaps, the so-called "star graphs" were invented to further enhance precisely this ability of line graphs. Unlike the cases of line graphs, however, it is the size and the shape of a polygon, rather than the paths of individual line segments, that convey information.

Look at the charts in Figure 3, borrowed from [4] with modifications. They display the overall performance of the highschool students in the regions A and B, measured by adjusted SAT score, unadjusted SAT score, and graduation rate. Here, information other than the individual values of these measures can be obtained from an inspection of the triangles appearing on the charts. Specifically, the obtuse angle of the triangle in the right graph indicates that B's school system has remarkable weakness in its graduation rate; the balanced but small triangle in the left graph indicates that the overall performance of A's school system is low, with no specific area of strength or weakness.

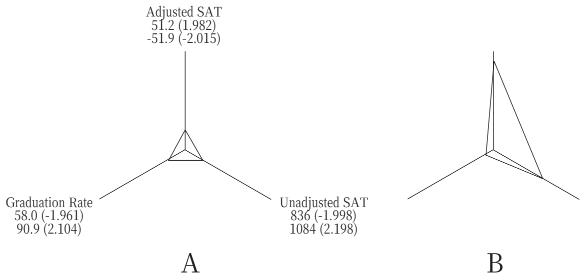

Figure 3. Star Graphs

Yet these informational relations do not seem to be a part of the basic semantic conventions for the star graphs. The basis semantic conventions specify the scales of each axis on a star plot, and thus determine what value is indicated by the locations of the polygon's vertex on each axis. The informational relations cited above are something that is derivable once these basic semantic relations are established.

3.3. Data Map

The phenomenon of derivative meaning is not confined to the display of numerical data in statistical charts. It is also relevant when we display spatial information in the form of a data map. For example, Tufte [12] cites John Snow, who used a street map of central London to plot the locations of deaths from cholera in September 1854. On his map in Figure 4, individual dots indicate the locations of deaths, and crosses indicate the area's eleven water pumps.

As with the case of scatter plots, the dots on the map form a cloud with a specific pattern and density, and it clearly indicates a general pattern of spatial distribution of cholera deaths. Specifically, the fact that many dots appear around the cross on the band denoting the Broad Street conveys the information that many deaths occurred around the water pump at the Broad Street.

Nevertheless, the informational relation between these two facts is a derivative one. The primitive semantic conventions consist of the usual semantic rules for the map, along with the specifications of the meanings of dots and crosses at particular locations on the map. Given these conventions, the particular pattern of the dots on the map *naturally* indicates a concentration of cholera deaths around the water pump at the Broad Street.

This derived informational relation played a historical role too: Snow used the information provided by it to find

Figure 4. A Data Map

out that the pump at the Broad Street was contaminated. He then had the handle of the pump removed and ended the neighborhood epidemic which had taken more than 500 lives. Tufte [12] says:

> Of course the link between the pump and the disease might have been revealed by computation and analysis without graphics, with some good luck and hard work. But, here at least, graphical analysis testifies about the data far more efficiently than calculation. (p. 24.)

3.4. Node-Edge Graph

A node-and-edge graph also supports derivative meanings. Think of a route map of the underground system in London, or of the subway system in any reasonably large city. Although such a route map typically gives a hint about the geographical locations of the displayed stations, it is essentially a node-edge graph. Thus, the basic semantic convention is that if an edge of a particular pattern or color connects two nodes, it means that the subway line denoted by that pattern or color connects the stations denoted by the nodes. For example, my route map of the London underground system has a dotted edge connecting the "Baker" node and the "Wall" node, and this means that the Pica Line connects the Baker Station and the Wall Station.

Given this semantic convention, it naturally holds that if two nodes are connected by a *path* consisting of edges with a single pattern or color, it means that there is a subway line connecting the stations denoted by the nodes. For example, my map of London's underground contains a path, consisting of three yellow edges, from the "Victoria" node to the "Embankment" node, and it means that the Victoria station is connected to the Embankment station by the Circle line. Note that this informational relation is not the same as the basic semantic conventions specified above: one refers to a *path* connecting two station nodes, while the other refers to an edge connecting two station nodes. The former is derivable from, but not the same as, the latter.

There seem to be several other informational relations derivable from the basic semantic convention for node-edge graphs, which seems to greatly contribute to the utility of this type of displays. For one thing, if a node in my route map has many edges touching it, it normally means that many different subway lines serve the station denoted by the node, connecting it to many different stations. Thus, the display of this type lets one easily detect the hubs of the city's subway system.

3.5. Table

Earlier, tables were contrasted to scatter plots and line graphs, as though tables never support the derivative informational relation that we have been discussing. This suggestion is wrong: some tables support a simple, but clear case of derivative meaning. Consider Table 2, which displays the functions of several models of low-cost printers.

	≤15W	≥1.8Mbps	A3	1440x720	USB	sublim
BJC430	○	○	×	×	×	×
PM700	○	×	×	×	×	×
PM750	○	×	×	×	×	×
PM750	○	○	○	○	○	×
PM2000	×	○	×	○	○	×
MD5000	×	×	○	×	×	○

Table 2. A Table

Here the basic semantic convention is that if a circle appears in the column representing the function X at the the row representing the printer model Y, it means that Y has the function X, while the appearance of a cross at the same position indicates the opposite. Given this basic rule, it naturally follows that the appearance of circles in *most* positions in the row labeled "PM750" means that the model PM750 has most of the functions listed in the table. It also follows that the appearance of *many* crosses in the column labeled "sublim" means that sublimination printing is a function rarely featured by the listed printers. There are probably many other informational relations derivable, but the point is that even a simple table such as Table 2 supports the phenomenon of derivative meaning, and that this fact greatly contributes the table's efficacy as a display of data.

4. Analysis

We have seen several examples of representation systems that support what may be called "derivative meaning," and found that its existence greatly explains the utility of those systems. We will now investigate how exactly this additional information relation arises in a given system. We take the simplest example, namely, the system of tabular representations just cited.

4.1. How Derivative Meaning Arises

For convenience, let us call this particular system $\mathcal{R}_t$. We write "$\bigcirc(n, m)$" to refer to the state of affairs that a circle appears in the n-th column at the m-th row, and "$\times(n, m)$" to refer to the appearance of a cross in that position. We also write "$F(n, m)$" to refer to the state of affairs that the printer model represented by the m-th row has the function represented by the n-th column, and write "$L(n, m)$" to refer to the opposite state of affairs. Thus, $\bigcirc(n, m)$ and $\times(n, m)$ are states of affairs that possibly hold in a table, while $F(n, m)$ and $L(n, m)$ are states of affairs that possibly hold in the printer-technology situation represented by a table.

With these notations, we can describe the basic semantic conventions for the system $\mathcal{R}_t$ in the following way:

(5) If $\bigcirc(n, m)$, then $F(n, m)$.

(6) If $\times(n, m)$, then $L(n, m)$.

For brevity, we write "$\bigcirc(n, m) \Rightarrow F(n, m)$" to mean (5), and "$\times(n, m) \Rightarrow L(n, m)$" to mean (6).

We had the intuition that the informational relation from (7) to (8) below is somehow derivable from these basic semantic conventions:

(7) Circles appear in most of the positions in the fourth row.

(8) PM750 has most of the functions listed in the table.

Let us assume that here the quantifier "most" is used with a definite sense, to mean "more than half."

Now (8) is an *abstract* state of affairs, in the sense that there are several alternative ways in which it is true. For example, one way in which (8) is true is that PM750 has five of the listed functions, namely, the first through the fifth. In our notation, we can conceive this "way" as the following collection of individual states of affairs:

$$\{F(1,4), F(2,4), F(3,4), F(4,4), F(5,4), L(6,4)\}$$

Here is another way in which (8) is true:

$$\{L(1,4), F(2,4), F(3,4), F(4,4), F(5,4), L(6,4)\}$$

Keep on this enumeration, and you eventually obtain the set of collections of this kind that exhausts all the alternative ways in which the state of affairs (8) holds. (Such a set exists since the quantifier "most" in (8) has a definite meaning.) Let Δ be this set. Then, if there is a member of Δ whose members are all true, this means that one of the sufficient conditions for (8) is satisfied, and hence (8) is true. Using $\bigwedge$ and $\bigvee$ to denote the operations of conjunction and disjunction on sets of states of affairs, we can express this fact in the following way:

(9) If $\bigvee\{\bigwedge \delta : \delta \in \Delta\}$, then (8) is true.

Note that a member of Δ is a collection of states of affairs about the printer market situation. We will now define the "corresponding" set Γ whose members are collections of states of affairs about the table itself.

Let γ be a set of states of affairs about the table, and δ be a set of states of affairs about the printer market. We say that γ is *projected to* δ if for each member γ_i of γ, there is a member δ_i of δ such that $\gamma_i \Rightarrow \delta_i$, and for each member δ_i of δ, there is a member γ_i of γ such that $\gamma_i \Rightarrow \delta_i$. For example, the following set is projected to the first of the two sets displayed above:

$$\{\bigcirc(1,4), \bigcirc(2,4), \bigcirc(3,4), \bigcirc(4,4), \bigcirc(5,4), \times(6,4)\}$$

We define Γ as the set of all collections γ such that γ is projected to some collection in Δ. Thus, each member of Γ has some member of Δ it is projected to, and each member of Δ has a member of Γ projected to it.

Given the implications (5) and (6), it logically follows that if there exists a member of Γ whose members are all true, then there exists a member of Δ whose members are all true. In other words, the following is true:

(10) If $\bigvee\{\bigwedge \gamma : \gamma \in \Gamma\}$, then $\bigvee\{\bigwedge \delta : \delta \in \Delta\}$.

To see this, assume the antecedent, and suppose that there is a member γ of Γ such that $\bigwedge \gamma$ is true. By the definition of Γ, γ is projected to some collection δ in Δ. Let δ_i be an arbitrary member of δ. By the definition of projection, $\gamma_i \Rightarrow \delta_i$ for some member γ_i of γ. But γ_i is true, since $\bigwedge \gamma$ is true. Since $\gamma_i \Rightarrow \delta_i$ and (5) and (6) are assumed, δ_i is true. Since each member of δ is shown to be true in this way, $\bigwedge \delta$ is true. Thus, $\bigvee\{\bigwedge \delta : \delta \in \Delta\}$ is true.

Now, it should be clear from the definition that Γ is the set of all the alternative ways in which more than three circles appear in the fourth row. Thus, Γ exhausts all the ways in which (7) is true. In other words, if (7) is true, there is some member of Γ whose members are all true. We can express this fact in the following way:

(11) If (7) is true, then $\bigvee\{\bigwedge \gamma : \gamma \in \Gamma\}$.

Combining (9), (10), and (11), we see that the informational relation from (7) to (8) obtains in fact. It is thus derivable, given the basic semantic conventions (5) and (6).

4.2. Conditions for Derivative Meaning

Exactly what feature of this system $\mathcal{R}_t$ of tabular representations makes this derivation possible? More generally, what conditions should a representation system satisfy to support such a derivation?

This answer becomes explicit when we consider the two ways in which one *could* extract information (8) from Table 2. The more tedious way is to inspect individual circles appearing in the fourth row in the table, observing the individual facts $\bigcirc(1,4)$, $\bigcirc(2,4)$, $\bigcirc(3,4)$, $\bigcirc(4,4)$, $\bigcirc(5,4)$, and $\times(6,4)$. According to the semantic conventions (5) and (6), these facts *respectively* indicate the facts $F(1,4)$, $F(1,4)$, $F(2,4)$, $F(3,4)$, $F(4,4)$, $F(5,4)$, and $L(6,4)$. These facts conjunctively imply the fact that PM750 has most functions listed in the table, and hence one could eventually extract the information (8).

Now, as we have seen, the system $\mathcal{R}_t$ provides an alternative way to extract this information. Surely the facts $F(1,4)$, $F(2,4)$, $F(3,4)$, $F(4,4)$, $F(5,4)$, and $L(6,4)$ conjunctively imply the information (8). But as we have seen, there is a much weaker state of affairs that implies (8), namely, $\bigvee\{\bigwedge \delta : \delta \in \Delta\}$. Now, given the basic semantic conventions for $\mathcal{R}_t$, $\bigvee\{\bigwedge \delta : \delta \in \Delta\}$ is in turn implied by the state of affairs $\bigvee\{\bigwedge \gamma : \gamma \in \Gamma\}$. What is special about the representation system $\mathcal{R}_t$ is the fact that it provides a state of affairs that entails $\bigvee\{\bigwedge \gamma : \gamma \in \Gamma\}$. The state of affairs is (7), and it is due to its existence at the beginning of this chain of entailments that the system $\mathcal{R}_t$ can provide an alternative indicator to the desired information (8).

$$
\begin{array}{lcl}
(7) & \vdash & \bigvee\{\bigwedge\gamma : \gamma \in \Gamma\} \\
& & \top \\
& & \bigvee\{\bigwedge\delta : \delta \in \Delta\} \quad \vdash \quad (8)
\end{array}
$$

Figure 5. A Chain of Entailments Supporting Derivative Meaning

Figure 5 gives a schematic review of this chain, where $\vdash$ (or $\top$) means an entailment. The bottom entailment from $\bigvee\{\bigwedge\delta : \delta \in \Delta\}$ to (8) is a constraint determined in the domain of represented objects, namely, the domain of printer market situations. The central entailment from $\bigvee\{\bigwedge\gamma : \gamma \in \Gamma\}$ to $\bigvee\{\bigwedge\delta : \delta \in \Delta\}$ is a logical consequence of the basic semantic conventions (5) and (6). The top entailment from (7) to $\bigvee\{\bigwedge\gamma : \gamma \in \Gamma\}$ is a constraint determined in the domain of representing objects, namely, the domain of tables on two-dimensional surfaces.

Our main claim is that whether a representation system supports a derivative meaning relation depends on the existence of a chain of entailment of this type. To be more exact, we characterize the conditions for derivative meaning in the following way:

Definition. Let $\mathcal{R}$ be a representation system. We say that a state of affairs α *indicates a state of affairs* β *derivatively in* $\mathcal{R}$ if there are sets Γ and Δ of collections of states of affairs such that:

1. $\alpha \vdash \bigvee\{\bigwedge\gamma : \gamma \in \Gamma\}$,
2. Each member of Γ is projected to some member of Δ, and each member of Δ has some member of Γ projected to it,
3. $\bigvee\{\bigwedge\delta : \delta \in \Delta\} \vdash \beta$.

Given the basic semantic conventions for a system $\mathcal{R}$, it is easy to see that if Γ and Δ satisfy clause 2, then $\bigvee\{\bigwedge\gamma : \gamma \in \Gamma\} \vdash \bigvee\{\bigwedge\delta : \delta \in \Delta\}$. Thus, under this condition, α in fact entails β, that is, the indication from α to β is an accurate one. Note that there is no guarantee for this entailment if the basic semantic conventions do not hold. This shows that the informational relation from α to β indeed depends on the basic semantic conventions.

4.3. Illustrating the Conditions

As the example of tabular representations illustrates, α in the above definition is often a global property of representations (such as dominant appearances of circles in a row) while members of Γ consist of more local properties (such as the exact position of an individual circle). Thus, clause 1 generally demands the existence of an entailment relation from a global property to local properties. In contrast, clause 3 generally demands an entailment of the opposite direction, from local properties (such as the possession of a specific function by a printer model) to a more global property (such as the possession of a majority of functions by a printer model).

Generally, our definition demands the existence of two kinds of constraints, namely, a *source* constraint on the structural properties of representations, which often goes from a global property to more local properties, and the *target* constraint on the represented objects, which often goes from local properties to a more global property.

Thus, in the examples in section 3, the source constraint takes the form of entailment (1) from a left slanted pattern of dots on a scatter plot to the positions of individual dots, (2) from a single exceptional leftward slope on a line graph to the exact positions of the end points of all slopes, (3) from the overall shape of a polygon on a star graph to the exact positions of the polygon's vertices, (4) from a cloud formed by dots on a data map to the positions of individual dots, and (5) from the path-level connections of station icons to their edge-level connections on a route map.

In contrast, the target constraint takes a form of entailment (1) from the X- and Y-values of individual data points to an overall correlation of X- and Y-values, (2) from the specific parafabuloid levels of various groups of people to the existence of an exceptional group, (3) from the graduation rates and the SAT scores of a group of students to the overall strength and weakness of the group, (4) from the locations of individual cholera deaths to their overall distributional tendency, and (5) from the immediate connections of London underground stations to their connections in the general sense.

Now our analysis claims that derivative meaning is generated when there is an additional constraint, directly ensured by semantic conventions, from the conseqeunt of a source constraint to the antecedent of the corresponding target constraint. Although we have no space to show that all these constraints hold in the individual cases, the above illustrations should make reasonably clear how our analysis applies to particular instances of derivative meaning.

5. Related Works

As partly documented in sections 1 and 3, the phenomenon of derivative meaning reported in this paper is often pointed out, and is even taken for granted, in the methodological studies on graphical data display such as [12, 5, 4]. Some of these works also report psychological studies on the ease in which humans perceives the graphical features carrying derivative meaning, as well as their effect on human performance in problem-solving. The topic, however, has not received sufficient treatment from the standpoints of formal semantics of graphical representations, and the exact mechanisms in which such derivative informational relations arise have been largely untouched.

Barwise and Etchemendy [1] initiated a series of works on model-theoretic semantics of graphical representations [10, 3, 6]. The framework has made possible rigorous investigations into the truth-conditions of graphical representations, and hence into the meta-logical properties, such as soundness and completeness, of deductive systems defined on graphical representation systems. As it stands, however, the framework lacks a mathematical device to express an entailment on the structural properties of graphical representations *per se*, as opposed to that on the properties of the represented objects. We have seen that an entailment of the first kind, namely, $\alpha \vdash \bigvee\{\bigwedge \gamma : \gamma \in \Gamma\}$ in our definition, is a critical condition for a system to support derivative meaning. Thus, our analysis implies that a significant extension of the mode-theoretic framework would be required, if it is to model the way in which a derivative semantic relation arises out of the basic semantic conventions.

The channel-theoretic framework used in [7] and further refined in [2] (chapter 20) does clearly distinguish these two kinds of entailments. In fact, the present author [7, 8] exploits this feature of the framework and characterizes several phenomena, such as free ride, overspecificity, autoconsistency, and constraint preservation, which have been argued to be specific to graphical systems of representations [9]. The framework appears to provide a appropriate setting for further formalizing the analysis developed in this paper.

Swoboda and Barwise [11] recently used the channel-theoretic framework to contrast information directly observable from graphics to information derivable only indirectly. Although the topic is clearly related to the one discussed in this paper, Swoboda and Barwise appealed to a different potential of channel theory than the one just cited. A comparison of our model to theirs is therefore an interesting project, but it is yet to be undertaken.

6. Conclusion

In conclusion, what is derivative meaning in graphical representations? What makes a feature α of a representation derivatively indicate a feature β of the represented in a given system? Briefly, the answer is the existence of a chain between α and β, consisting of three links of different kinds. The first link is an entailment determined in the domain of the representing objects. The middle link is an entailment ensured by the basic semantic conventions for the system. And the third link is an entailment determined in the domain of the represented objects. Thus, intuitively, a derivative meaning relation is the result of extending the basic semantic relation (the middle link) with a constraint on the representing objects (the first link) and a constraint on the represented objects (the third link). Some systems support such an extension, and some don't. Those that do support generally enjoy the enrichment of the semantic contents of the individual representations that belong to them. The scatter plots, the line graph, the data map, the node-edge graph, and the table encountered in this paper are all instances of such enriched representations.

References

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